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Testing Reliability Equivalence Factors of a Series-Parallel Systems in Burr Type X Distribution

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Abstract

In reliability analysis for improving the system performance, the scale parameter of the life time model has mainly considered to obtain equivalence factors for the system designs. In this paper, we propose a new approach through modifying the shape parameter of the Burr type X distribution. The proposed approach is applied to the general series parallel systems. Three different methods are used to improve the system reliability: (i) the reduction method, (ii) the hot duplication method and (iii) the cold duplication method. Numerical example is presented to compare performance of the applied methods, to find limitations for the equivalence factors and to illustrate the overall theoretical analysis.

Keywords: Reliability, series-parallel system, equivalence factor, burr type x distribution, hot and cold duplications.

1 Introduction

Reliability evaluation is an important and integral part for developing most of the engineering systems. Operation researches in reliability theory are mainly concerned with the problem of having a system perform in the best possible way. Generally the system performance can be improved using standby redundancy methods. Particularly the most commonly used of such methods are:

1. Hot duplication method: in this method, it is assumed that some of the system components are duplicated in parallel.
2. Cold duplication method: in this method, it is assumed that some of the system components are duplicated in parallel via a perfect switch.

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For many different reasons, such as space limitation, and high manufacturing costs, using the redundancy methods may not be the optimal solution for the system in which the minimum size and weight are overriding consideration; for example, in satellites or other space applications, in well logging equipment, and pacemakers as similar to biomedical applications [1]. In such applications space or weight limitations may indicate an increase in component reliability rather than redundancy. Then emphasis must be placed on robust design controlling the operation environment. Since it is not always possible to improve a system by duplicating some or all of its components, engineers have adopted the reduction method. In this method, it is assumed that the failure rates of some of the system components are reduced by a factor $\rho\ , 0 < \rho < 1$.In applying the reduction method, the main recurrent problem is the level for which the failure rate should be decreased to achieve the equivalence performance as applying the duplication methods.

Rade [2] introduced the concept of reliability equivalence through his researches in statistical quality controlling. He defined the reliability equivalence factor as a factor by which a characteristic of system design has to be multiplied in order to reach equality of a characteristic for a different standard design. Equivalently, Sarhan [3] has used the reliability function as the performance measure of the system reliability.

Beyond the assumptions of constant failure rates modeling by the Exponential Distribution, Xia and Zhang [4] considered equivalence factors in Gamma distribution. El-Damcese [5] obtained the reliability equivalence factors of series-parallel systems in the Weibull distribution. Mustafa and El-Faheem [6] found the reliability equivalence factors of a general parallel system with mixture of life time distributions. Also, Shawky et al. [7] considered the reliability equivalence for the Exponentiated Exponential distribution. In the previous mentioned studies, the hazard and the reliability functions are decreases or increases through the indexed scale parameter. In reliability general frame analysis, there exists other lifetime distributions for which the hazard and reliability functions are not affected by the scale parameter, and mainly affected by the shape parameter.

Burr type X distribution initially proposed by Burr [8] and investigated as a generalization of the Rayleigh distribution by Mudholkar and Srivastava [9]. This distributionis effectively modeled in general lifetime data and considered by many authors as Jaheen [10], Ahmad et al. [11], Aludaat et al. [12], El-Damcese and Ayoub [13] and Migdadi and Al-Batah [14].

The cumulative distribution of the random variable T having the Burr type X distribution is given by

$$F(t;\lambda,\theta) = (1 - e^{-\lambda t^2})^{\theta} \qquad ,t > 0\ ,\lambda\ ,\theta > 0 \tag{1}$$

This implies, the reliability function is given by

$$R(T;\lambda,\theta) = 1 - (1 - e^{-\lambda t^2})^{\theta} \qquad ,t > 0\ ,\lambda\ ,\theta > 0 \tag{2}$$

The probability distribution and the hazard functions are given respectively by

$$f(t;\lambda,\theta) = 2\theta\lambda e^{-\lambda t^2}(1 - e^{-\lambda t^2})^{\theta-1} \qquad ,t > 0\ ,\lambda\ ,\theta > 0 \tag{3}$$

$$h(t;\lambda,\theta) = \frac{2\theta\lambda e^{-\lambda t^2}(1-e^{-\lambda t^2})^{\theta-1}}{1-(1-e^{-\lambda t^2})^{\theta}} \qquad ,t > 0\ ,\lambda\ ,\theta > 0 \tag{4}$$

where θ is the shape parameter and λ is the scale parameter.

Surles and Padgett [15] showed that Burr type X distribution can be used quite effectively in modeling strength data. It was also shown by Raqab and Kundo [16] that the hazard rate and the reliability functions of Burr type X distribution is not affected by the scale parameter λ and the hazard rate function is a bathtub type for $\theta \leq 0.5$ and it is an increasing function for $\theta > 0.5$.This flexibility in the hazard rate function makes burr type X a probable model for many reliability systems.

In this paper, the reliability equivalence factors for the general series-parallel system in the Burr type X distribution are derived using the reliability function through modifying the shape parameter θ by multiplying with a factor, $\rho > 0$.

The rest of this paper is organized as follows. In section 2, the reliability and the mean time to failure (MTTF) of the general series-parallel system are derived. In section 3, the reliability and MTTF of the improved system using the reduction and the standby (hot and cold) are obtained. The reliability equivalence factors for both hot and cold duplications are obtained in section 4. Numerical results for the theoretical findings are presented through an illustrative Numerical example in section 5. Limitations of the study and highlights for future work are included in section 6.Finally general conclusion of the overall study is implemented in section 7.

2 Series-Parallel Systems

The system considered here, consists of m subsystem connected in parallel, with subsystem i consisting of n_i components connected in series for$i = 1,2,\dots,m$. (Fig. 1) shows the diagram of a series-parallel system.

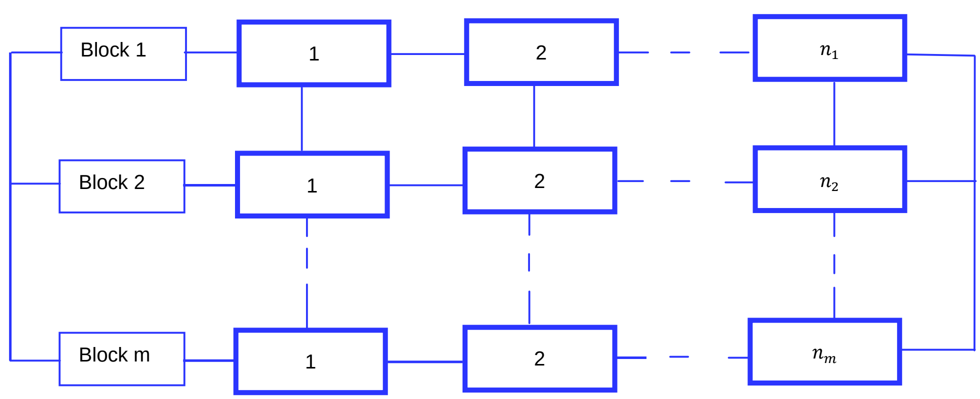

Fig. 1. General series-parallel system

Let $R_i(t)$ be the reliability of subsystem i and $r_{ij}(t)$ be the reliability of component$j$, $1 \leq j \leq n_i$in subsystem, $i = 1,2,\dots,m$. Then

$$R_i(t) = \prod_{j=1}^{n_i} r_{ij}(t) \tag{5}$$

This implies, the system reliability is given by

$$R_s(t) = \prod_{i=1}^{m} (1 - R_i(t))$$

Using (5), the system reliability is

$$R_s(t) = 1 - \prod_{i=1}^{m} (1 - \prod_{j=1}^{n_i} r_{ij}(t)) \quad (6)$$

Assuming that, the system components are independent and identical having the Burr type X distribution. This implies that

$$r_{ij}(t) = 1 - (1 - e^{-\lambda t^2})^{\theta}\,, \qquad j = 1,2,\dots,n_i, i = 1,2,\dots,m$$

Hence, the system reliability becomes

$$R_s(t) = 1 - \prod_{i=1}^{m} (1 - (1 - (1 - e^{-\lambda t^2})^{\theta})^{n_i}) \quad (7)$$

Using equation (7), the system mean time to failure (MTTF) can be derived in the following form [4].

$$MTTF = \int_0^{\infty} R_s(t) dt \quad (8)$$

3 The Improved System

In this section, the reliability functions and the MTTF of the improved system according to the reduction and standby redundancy “hot and cold” are derived.

3.1 The Reduction Method

In this method, it is assumed that the reliability of k_i identical components of the subsystem $i\,, i = 1,2,\dots,m$is improved by increasing the reliability function through multiplying the shape parameter by a factor$\rho\ \ , \rho > 0$.Therefore, using (2), the reliability of each of the k_i components of the subsystem $i\,, i = 1,2,\dots,m$ is given by

$$r_{red}(t) = 1 - (1 - e^{-\lambda t^2})^{\rho\theta} \quad (9)$$

setting: $X = (1 - e^{-\lambda t^2})$ (10)

This implies, the reliability of the system improved by the reduction method is given by

$$R_{red}(t) = 1 - \prod_{i=1}^{m} (1 - \left(1 - X^{\rho\theta}\right)^{k_i} \left(1 - X^{\theta}\right)^{n_i - k_i}) \quad (11)$$

Therefore, the MTTF of the system becomes

$$MTTF = \int_0^{\infty} R_{red}(t)dt \tag{12}$$

3.2 Hot Duplication Method

In this method, it is assumed that some of the system components are duplicated in parallel. If h_i , $i = 1,2,\dots,m$ components are hot duplication, the reliability for each of the h_i , $i = 1,2,\dots,m$ components is given by

$$r_h(t) = (2 - \left(1 - \left(1 - e^{-\lambda t^2}\right)^{\theta}\right)\left(1 - \left(1 - e^{-\lambda t^2}\right)^{\theta}\right) \tag{13}$$

Using (10) and (2), the reliability function of the system improved by the hot duplication method becomes

$$R_h(t) = 1 - \prod_{i=1}^{m}(1 - \left(1 - X^{2\theta}\right)^{h_i}\left(1 - X^{\theta}\right)^{n_i - h_i}) \tag{14}$$

Therefore, the MTTF of the system becomes

$$MTTF = \int_0^{\infty} R_h(t)dt \tag{15}$$

3.3 Cold Duplication Method

In this method, some of the system components are duplicated in parallel via a perfect switch. Following Rade [2], the reliability function of each component improved by a cold via perfect switch can be given by

$$r_C(t) = r(t) + \int_0^{t} f(y)r(t-y)dy \tag{16}$$

where $r(t)$ is the reliability function of the component in the original system.

If w_i units from subsystem $i = 1,2,\dots,m$is improved by the cold duplication, then using (2), (3), and (10), the reliability function of the system becomes

$$R_C(t) = 1 - \prod_{i=1}^{m}(1 - Q(Y,\lambda,\theta)^{w_i}\left(1 - X^{\theta}\right)^{n_i - w_i}) \tag{17}$$

where, $Q(Y,\lambda,\theta) = \int_0^{t} ye^{-\lambda y^2}(1 - e^{-\lambda y^2})^{\theta-1}(1 - (1 - e^{-\lambda(t-y)^2})^{\theta})dy$

The above integral can be numerically evaluated. Therefore, the MTTF of the system becomes

$$MTTF = \int_0^{\infty} R_C(t)dt \tag{18}$$

4 Reliability Equivalence Factors

In this section, the reliability equivalence factors of the improved systems are derived. The reliability equivalence factor(s) denoted by $\rho_{(\alpha)}^{D}$, $D = H,(C)$ for hot, (cold) duplication is defined as that factor ρ by which the failure rate for the set of system components should be reduced,or equivalently the reliability function increased so that one could obtain a design of the system with a reliability function of a design obtained from the original system. For the hot duplication,$\rho_{(\alpha)}^{h}$can be obtained by solving the set of the two equations

$$R_{red}(t) = \alpha \,, \qquad R_{hot}(t) = \alpha$$

Substituting for $R_{red}(t)$, and $R_{hot}(t)$from equation (11) and (14),$\rho_{(\alpha)}^{h}$can be obtained by solving the following two nonlinear equations

$$R_{red}(t) = 1 - \prod_{i=1}^{m}(1 - \left(1 - X^{\rho_{(\alpha)}^{h}\theta}\right)^{k_i}\left(1 - X^{\theta}\right)^{n_i - k_i}) = \alpha \tag{19}$$

$$R_h(t) = 1 - \prod_{i=1}^{m}(1 - \left(1 - X^{2\theta}\right)^{h_i}\left(1 - X^{\theta}\right)^{n_i - h_i}) = \alpha \tag{20}$$

with respect to X and $\rho_{(\alpha)}^{h}$

Similarly, for the cold duplication $\rho_{(\alpha)}^{c}$ can be obtained by solving the set of the two equations

$$R_{red}(t) = \alpha, \qquad R_c(t) = \alpha$$

Substituting for $R_{red}(t)$, and $R_C(t)$ from equation (11) and (17), $\rho_{(\alpha)}^{c}$ can be obtained by solving the following two nonlinear equations

$$R_{red}(t) = 1 - \prod_{i=1}^{m}(1 - \left(1 - X^{\rho_{(\alpha)}^{c}\theta}\right)^{k_i}\left(1 - X^{\theta}\right)^{n_i - k_i}) = \alpha \tag{21}$$

$$R_C(t) = 1 - \prod_{i=1}^{m}(1 - Q(Y, \lambda, \theta)^{w_i}\left(1 - X^{\theta}\right)^{n_i - w_i}) = \alpha \tag{22}$$

With respect to X and $\rho_{(\alpha)}^{c}$

5 Numerical Results

To illustrate the theoretical results obtained in the previous sections. Some numerical results are given in the following example:

Consider a series-parallel system with $n = 5$ units distributed in $m = 2$ subsystems where $n_1 = 2$ units in the subsystem 1 and $n_2 = 3$ units in the subsystem 2. Assume the scale parameter is fixed to be$= 3$, and the components are independent and Burr type X identically distributed.

For the values of $\alpha = 0.1$,0.5 , 0.9 , and $(k_1, k_2) = (0,1), (1,0), \dots, (2,3)$, where k_1, k_2 are the number of units in the subsystems 1 and 2 respectively that are improved by the reduction method. $\rho_{(\alpha)}^{h}$: The reliability equivalence factor for the hot duplication method is obtained for different configurations of $(h_1, h_2) = (0,1), (1,0) \dots, (2,3)$, where h_1, h_2 are the numbers of units in the subsystems 1 and 2 respectively that are improved by the hot duplication method as in (Table 1). Similarly$\rho_{(\alpha)}^{c}$: The reliability equivalence factor for the cold duplication method is obtained for different configurations of $(w_1, w_2) = (0,1), (1,0), \ldots, (2,3)$, where w_1, w_2 are the numbers of units in the subsystems 1 and 2 respectively that are improved by the cold duplication method as in (Table 2). Values (- -) of $\rho_{(\alpha)}^{D}$, $D = H, (C)$ for hot, (cold) duplications means that it is not possible to improve the design of the system to be equivalent with the design of the system which can be obtained by improving the components according to hot or cold duplications. Figs. 2 and 3 represent the reliability functions of the systems improved by hot and cold duplications at different settings of $(h_1, h_2), (w_1, w_2)$.

Table 1. $\boldsymbol{\rho_{(\alpha)}^{h}}$: The reliability equivalence factor for the hot duplication method

$(\mathbf{h_1}, \mathbf{h_2})$	$\boldsymbol{\alpha}$	$(\boldsymbol{k_1}, \boldsymbol{k_2})$										
		(0,1)	**(0,2)**	**(0,3)**	**(1,0)**	**(1,1)**	**(1,2)**	**(1,3)**	**(2,0)**	**(2,1)**	**(2,2)**	**(2,3)**
(0,1)	0.1	2	1.985	1.521	1.473	1.387	1.276	1.271	1.258	1.252	1.203	1.107
	0.5	2	1.950	1.331	1.322	1.320	1.195	1.132	1.251	1.249	1.117	1.092
	0.9	2	1.321	1.292	1.240	1.231	1.191	1.130	1.107	1.061	1.012	1.011
(0,2)	0.1	--	2	1.733	--	1.725	1.681	1.601	1.582	1.524	1.427	1.399
	0.5	--	2	1.562	--	1.710	1.485	1.483	1.461	1.361	1.291	1.256
	0.9	--	2	1.453	--	1.556	1.450	1.329	1.328	1.320	1.280	1.205
(0,3)	0.1	--	--	2	--	--	--	1.962	1.831	1.758	1.728	1.682
	0.5	--	--	2	--	--	--	1.795	1.780	1.680	1.551	1.483
	0.9	--	--	2	--	--	--	1.782	1.756	1.570	1.493	1.532
(1,0)	0.1	--	1.982	1.807	2	1.753	1.658	1.608	1.574	1.523	1.496	1.375
	0.5	--	1.965	1.502	2	1.630	1.450	1.388	1.385	1.321	1.276	1.238
	0.9	--	1.510	1.238	2	1.542	1.220	1.217	1.216	1.215	1.195	1.150
(1,1)	0.1	--	--	1.825	--	2	1.789	1.652	1.608	1.592	1.552	1.502
	0.5	--	--	1.650	--	2	1.648	1.451	1.501	1.450	1.360	1.310
	0.9	--	--	1.521	--	2	1.523	1.430	1.426	1.421	1.323	1.251
(1,2)	0.1	--	--	1.962	--	--	2	1.768	1.745	1.682	1.622	1.601
	0.5	--	--	1.950	--	--	2	1.655	1.720	1.630	1.541	1.449
	0.9	--	--	1.721	--	--	2	1.492	1.681	1.552	1.480	1.370
(1,3)	0.1	--	--	--	--	--	--	2	--	1.961	1.902	1.834
	0.5	--	--	--	--	--	--	2	--	1.938	1.797	1.656
	0.9	--	--	--	--	--	--	2	--	1.921	1.850	1.650
(2,0)	0.1	--	--	--	--	--	--	--	2	1.987	1.901	1.814
	0.5	--	--	--	--	--	--	1.966	2	1.901	1.765	1.635
	0.9	--	--	--	--	--	--	1.798	2	1.796	1.689	1.519
(2,1)	0.1	--	--	--	--	--	--	--	--	2	1.973	1.825
	0.5	--	--	--	--	--	--	--	--	2	1.886	1.689
	0.9	--	--	--	--	--	--	--	--	2	1.794	1.598
(2,2)	0.1	--	--	--	--	--	--	--	--	--	2	1.947
	0.5	--	--	--	--	--	--	--	--	--	2	1.834
	0.9	--	--	--	--	--	--	--	--	--	2	1.778
(2,3)	0.1	--	--	--	--	--	--	--	--	--	--	2
	0.5	--	--	--	--	--	--	--	--	--	--	2
	0.9	--	--	--	--	--	--	--	--	--	--	2

Table 2. $\rho^{c}_{(\alpha)}$: The reliability equivalence factor for the cold duplication method

$(\mathbf{h_1},\mathbf{h_2})$	$\boldsymbol{\alpha}$	$(\boldsymbol{k_1},\boldsymbol{k_2})$										
		(0,1)	**(0,2)**	**(0,3)**	**(1,0)**	**(1,1)**	**(1,2)**	**(1,3)**	**(2,0)**	**(2,1)**	**(2,2)**	**(2,3)**
(0,1)	0.1	2	1.675	1.321	1.273	1.209	1.175	1.161	1.158	1.154	1.103	1.008
	0.5	2	1.552	1.316	1.227	1.121	1.165	1.135	1.131	1.117	1.017	1.005
	0.9	2	1.318	1.274	1.213	1.122	1.120	1.118	1.105	1.043	1.011	1.001
(0,2)	0.1	--	2	1.483	--	1.426	1.371	1.322	1.286	1.128	1.224	1.219
	0.5	--	2	1.461	--	1.417	1.295	1.281	1.253	1.121	1.203	1.157
	0.9	--	2	1.352	--	1.358	1.252	1.223	1.119	1.110	1.080	1.075
(0,3)	0.1	--	--	2	--	--	--	1.467	1.401	1.356	1.321	1.304
	0.5	--	--	2	--	--	--	1.398	1.385	1.370	1.256	1.243
	0.9	--	--	2	--	--	--	1.361	1.310	1.275	1.192	1.176
(1,0)	0.1	--	1.780	1.583	2	1.557	1.482	1.410	1.474	1.343	1.299	1.271
	0.5	--	1.662	1.492	2	1.434	1.387	1.382	1.378	1.320	1.264	1.250
	0.9	--	1.417	1.378	2	1.341	1.220	1.207	1.202	1.115	1.107	1.102
(1,1)	0.1	--	--	1.825	--	2	1.584	1.451	1.408	1.322	1.259	1.207
	0.5	--	--	1.650	--	2	1.548	1.351	1.302	1.258	1.190	1.115
	0.9	--	--	1.521	--	2	1.421	1.270	1.218	1.201	1.162	1.111
(1,2)	0.1	--	--	1.962	--	--	2	1.503	1.441	1.387	1.320	1.203
	0.5	--	--	1.950	--	--	2	1.485	1.422	1.351	1.312	1.144
	0.9	--	--	1.721	--	--	2	1.392	1.301	1.257	1.232	1.129
(1,3)	0.1	--	--	--	--	--	--	2	--	1.666	1.502	1.432
	0.5	--	--	--	--	--	--	2	--	1.537	1.495	1.358
	0.9	--	--	--	--	--	--	2	--	1.424	1.358	1.275
(2,0)	0.1	--	--	--	--	--	--	--	2	1.782	1.651	1.517
	0.5	--	--	--	--	--	--	1.966	2	1.603	1.468	1.431
	0.9	--	--	--	--	--	--	1.798	2	1.591	1.382	1.311
(2,1)	0.1	--	--	--	--	--	--	--	--	2	1.973	1.825
	0.5	--	--	--	--	--	--	--	--	2	1.886	1.689
	0.9	--	--	--	--	--	--	--	--	2	1.794	1.598
(2,2)	0.1	--	--	--	--	--	--	--	--	--	2	1.947
	0.5	--	--	--	--	--	--	--	--	--	2	1.834
	0.9	--	--	--	--	--	--	--	--	--	2	1.778
(2,3)	0.1	--	--	--	--	--	--	--	--	--	--	2
	0.5	--	--	--	--	--	--	--	--	--	--	2
	0.9	--	--	--	--	--	--	--	--	--	--	2

From Tables 1 and 2, it appears clearly that:

1. $\rho^{c}_{(\alpha)}$, $\rho^{h}_{(\alpha)}$ decreases as α increases for different settings of $(h_1, h_2), (w_1, w_2)$.
2. For fixed values of $(h_1, h_2), (w_1, w_2)$, $\rho^{h}_{(\alpha)}$, $\rho^{c}_{(\alpha)}$ are decreases as the number of improved units according to the reduction method (k_1, k_2) increases.
3. It is not possible to improve the system with values of $\rho^{c}_{(\alpha)}$, $\rho^{h}_{(\alpha)} < 1$.
4. It is not possible to obtain equivalence factors with values greater than 2.

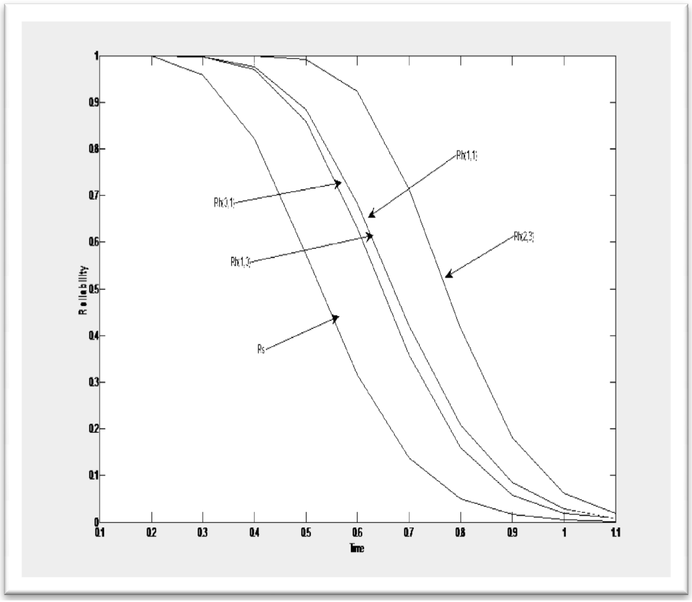

Fig. 2. Reliability for the system improved by hot duplication of (0, 1), (1, 0), (1, 1), (2, 3) from the subsystems 1 and 2 respectively and Rs: The reliability of the original system

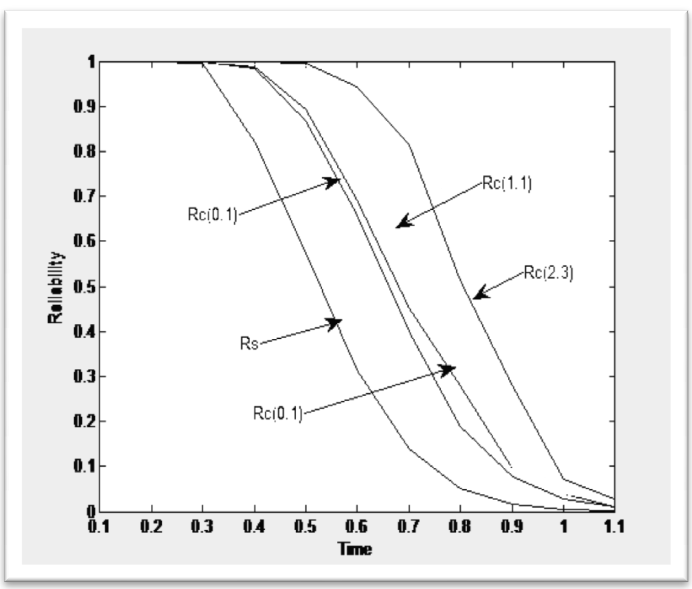

Fig. 3. Reliability for the system improved by cold duplication of (0, 1), (1, 0), (1, 1), (2, 3) from the subsystems 1 and 2 respectively, and Rs: The reliability of the original system

From Figs. 2 and 3, we find that

1) Hot duplication of (0,1) components gives an improved design with lowest reliability function among all of other improved designs which can be obtained by improving any other settings of components according to either hot or cold duplications.
2) Hot duplication of (0,1) components gives an improved design with lowest reliability function among all of other improved designs which can be obtained by improving any other settings of components according to either hot or cold duplications.
3) At level of reliability 0.6, the MTTF increases from 0.48 time units to 0.59 time units by improving (0,1) components, to 0.615 time units by improving (1,1) components and to 0.713 time units by improving (2,3) components according to hot duplication.
4) At level of reliability 0.6, the MTTF increases from 0.48 time units to 0.62 time units by improving (0,1) components, to 0.64 time units by improving (1,1) components and to 0.78 time units by improving (2,3) components according to cold duplication.

6 Limitations and Future Work

In this study, the shape parameter is investigated to obtain reliability equivalence factors for the general series parallel systems in Burr type X distribution. This is reasonable, since the hazard rate and the reliability functions of Burr type X distribution are mainly affected by the shape and not the scale parameter because the hazard rate function is increasing when the shape parameter $\theta > 0.5$. This consideration may be generalized to more other bathtub hazards lifetimes models which are frequently used in reliability analysis. However, there exists many other life time distributions for which neither the hazard rate nor the reliability functions are affected by the shape parameter, and hence, modifying the scale parameter and not the shape parameter will be the subject of concern. Future work may highlights other configurations of the system designs, like non-identical system units, mixed system units, k out of n systems and complex configurations systems. The repairing and the maintenance operating processes can also be implemented using different transition markovian approaches. Reliability equivalence for discrete lifetime's distributions may also be a subject of interest. The analysis of censored and interval censored data could also be included using Bayesian analysis approaches.

7 Conclusion

In this paper, the reliability function is used through modifying the Burr type X shape parameter as a performance measure to compare the system reliability of different designs. The reliability equivalence factors of the general series-parallel systems in Burr type X distribution are obtained. Three different methods are used to improve the system reliability: (i) the reduction method, (ii) the hot duplication method and (iii) the cold duplication method. Numerical results for different system configurations manifest the performance of the theoretical findings and indicate levels for the reliability equivalence factors: $\rho^{c}_{(\alpha)}$, $\rho^{h}_{(\alpha)}$obtained by either cold or hot duplications to be $1 \leq \rho^{c}_{(\alpha)}$, $\rho^{h}_{(\alpha)} \leq 2$.Limitations for this study are restricted for lifetime's models for which the hazard and the reliability functions are mainly affected by the shape parameter. Future studies may extends by considering a more general class of lifetimes models through reliability and survival analysis issues using different statistical inferential methods.

Competing Interests

Authors have declared that no competing interests exist.

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