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Chaos and Boundary Values Problems of Mathematical Models of Nonautonomous Dynamical Systems

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ABSTRACT

The boundary values of problem that are determined from observations play a decisive role in solving any problem in mathematical models of dynamic systems.
They lead to the search of answer to the following questions:
From observations, is it possible to find such boundary values, which could become the guarantor of the existence of smooth or chaotic solutions of the problem?
This paper presents estimates of variations calculated from numerous observations: border estimates of the variations of the gravitational constant of the solar system:

$$10^{-13} \le \dot{G}/G \le 10^{-11}\, year^{-1}; \qquad \text{(I)}$$

Border estimates of the variations of iovocentric coordinates of V satellite of Jupiter:

$$0'',22 \le \xi\,(0-C) \le 0''81, \quad -0'',61 \le \eta\,(0-C) \le 0''54; \qquad \text{(II)}$$

border estimates of the variations of drift motions of daily satellites' major semi-axes for the points where they intersect with the equator, vary within:

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$$-4{,}2\,km/day \quad \leq \Delta\dot{\rho} \leq 4{,}2\,km/day \tag{III}$$

border estimates of changes of daily satellites' longitude vary within:

$$-0{,}0053\ \text{deg.}/day \leq \Delta\dot{\lambda} \leq 0{,}0045\ \text{deg.}/day \tag{IV}$$

border estimates of the variations of the rotation period of the daily satellites vary within:

$$-0{,}053 \leq \Delta T \leq 0{,}055\ day \tag{V}$$

Ratings (I)-(V) obtained from the analysis of sums of infinitesimal perturbations of range less than or equal to the errors of observation.
When satisfying the boundary estimates (I) - (V) of Solar System, orbits of V satellite and daily satellites are stable. Once these conditions are violated, chaos creeps in the orbits of the Solar System, V satellites and daily satellites and the orbits become unstable.

Keywords: Boundary value of problems; chaos; mathematical modeling; dynamical systems.

1. INTRODUCTION

The processes observed in nature are studied by observation. Values of observations are made up of three components: 1) the sum of the main forces under the influence of which processes are formed and take place. They can be studied by using methods of mathematical modeling. 2) the sum of infinitely many and small forces that subtly and covertly take part in all of the observed processes and phenomena. They often show themselves in the form of resonance phenomena, i.e., as chaos and catastrophe. 3) measures of inaccuracy of the auxiliaries used. They do not affect the course of events, are constant and do not depend on time.

This paper presents a spectral analysis of the sum of infinitely many and small actions (forces, perturbations) that can't be captured and that are covertly take part in all processes and phenomena. In some resonant cases, they often cause chaos and disasters.

2. THE BOUNDARY VALUES OF OBSERVATIONS AND CHAOS

In this paper, we consider the chaos and the boundary values in the differential equations of mathematical models of dynamic systems, which describe the motion of a particle in a non-autonomous field of gravitational forces.

Generation, formation and evolution of complex dynamic systems and also their structural elements depend on the initial values of a variety of restrictions on the parameters of systems, i.e. boundary value problems [1-3].

The disclosure of a set of observed geometric and dynamic phenomena and processes is related to the accuracy of measuring devices, i.e. the accuracy of the observational data. They reflect both the immediate impact and changes of all the internal structures and external variations of the shape of the Earth and other objects. The data sharpening is only possible through improvement of the accuracy of measuring devices and their different combinations, including the accuracy of measurement of observation time [4-6,7,8,9,10].

The sums of implicit infinitesimal perturbations are included in the sums of infinitesimal estimates of hidden errors.

In all of the observed phenomena and processes, there are always non-autonomous hidden nonlinear sums of small perturbations. They can be cyclic, periodic, almost periodic or random by nature, without any signs of cycle or periodicity over the time [9,10,11].

It is known that [5,12,8,13,14] the movement of some objects in the field of non-autonomous–non-central forces can be represented by the differential equation of the form:

$$\varepsilon = \frac{d^2x}{dt^2} - \varphi\left[\frac{dx}{di}, g(x), t\right] = \mu R(x,t) \tag{1}$$

where $x \in (x, x_{i+1}),\ t \in (t_i, t_{i+1}),\ i = \overline{1, n};$

$\varphi\left[\frac{dx}{dt}, g(x), t\right]$ -represents the sum of the effects that allow mathematical modeling i.e. the part of the amount of exposure that allows to solve the problem in closed form; $\mu R(x,t) -$ amount of elusive-hidden disturbances which can cause all sorts of chaos, (μ-small parameter of the problem) . They are usually equal to or less than the error of observations.

Solutions of differential equations (1), in general, can be presented without any signs of cycle or periodicity, through boundary value problems by the system of the following equations:

$$\left.\begin{array}{l} x(t-1) = f[x(t-2)], \\ x(t) = f[x(t-1)], \\ x(t+1) = f[x(t)],\ t > 0. \end{array}\right\}. \quad (2)$$

The system (2) allows a solution of differential equations (1) using the model problems, in particular, the model problem of three fixed centers [15]. The dynamical systems, differential equations representing them and mathematical models combine boundary values (conditions) of the problem. Each set of the numerous of boundary value problems is highlighted with their particular solutions of the problem.

Based on the observations, the selection of boundary values of the problem is usually limited to three objectives:

1) find such boundary values(conditions) of problem from observations that could be guarantor of the existence of a general solution of the problem;
2) find the boundary values of the observations, which would serve as a guarantor of the existence of chaos. They allows to determine the time, place and the forces of chaos;
3) find the boundary values of problem from the observations for which corresponding solutions of differential equations of the problem can be guarantor of the existence of chaos. They allow you to define the time, place and the forces of chaos.

The equations of motions in non-holonomic nonlinear dynamic systems can be presented as:

$$\begin{array}{l} \varepsilon = \dfrac{dx}{dt} - f(x,y) = \mu P(x,y,t); \\ \varepsilon = \dfrac{dy}{dt} - g(x,y) = \mu Q(x,y,t) \end{array} \quad (3)$$

where $(x, y) \in R^2$ phase coordinates, $f(x, y)$ and $g(x, y)$ allows mathematical modeling of the problem, μ-the small parameter of the problem, $P(x, y, t)$ and $Q(x, y, t)$ subtle perturbations, numerical values of which are less than or equal to the errors of observations.

Boundary values of the problems defined from the observations combine real dynamical systems, their differential equations and various mathematical models.

In mathematical models, the set of parameters, found from observations, can be numerous. Some of them correspond to the elusive area of the boundary values of time, place (position) and the forces of chaos. These are: tsunamis, earthquakes, landslides, volcanic eruptions, numerous atmospheric phenomena, etc. Finding of the relevant hidden boundary values are among the most pressing problems.

It should be noted that in [16,8,17] the evolution of the amount of small perturbations, the order of the observational errors have been explored in a graphical manner. In the examples to determine the accuracy of the gravitational constant G and ξ and η of iovocentric coordinates of V Jupiter's moon, we received:

$$10^{-13} \le \varepsilon\left[\dot{G}/G\right] \le 10^{-11}\, year^{-1}; \quad (4)$$

$$-O''22 \le \varepsilon\ \left[\xi(O-C\ )\right] \le O''81; \quad (5)$$

$$-O''61 \le\ \varepsilon\left[\eta\ (O-C)\right] \le O''54; \quad (6)$$

Where *O*- is the values of observations and *C*- is the values of calculations.

In the boundary values (4), (5), (6) ε indicate that they correspond to the variations of the elusive-hidden borders of the amounts of small perturbations of the degree of the errors in accordance with perturbation theory of A.M. Lyapunov [18].

Condition (4) indicates the stability of the Solar System, in the sense of Lyapunov. When these conditions are violated, the chaos is creeping in the Solar System and it becomes suns table.

Conditions (5) and (6) are the conditions of stability of the orbit of Jupiter's satellite V. When these conditions are violated, the chaos is creeping in the orbit of V satellite and it becomes unstable in the sense of Lyapunov.

The terms of the stability of orbits of daily satellites received in [8]. They are as follows:

Drift motions of major semi-axes of daily satellites for the points where they intersect with the equator, vary within:

$$-4{,}2\,km/day \le \varepsilon(\Delta\dot{\rho}) \le +4{,}2\,km/day \qquad (7)$$

Under the influence of the amount of small perturbations, the longitudes of daily satellites vary within the following range:

$$-0{,}0053\ \deg./day \le \varepsilon(\Delta\dot{\lambda}) \le 0{,}0045\ \deg./day \qquad (8)$$

Under the influence of the amount of small perturbations the rotation period of stationary satellites varies:

$$-0{,}0053\,day \le \varepsilon(\Delta \mathrm{T}) \le 0{,}055\,day \qquad (9)$$

The limits of the estimates (7), (8), (9) obtained from the analysis of orbits of stationary satellites "Syncom-2", "Syncom-3" and "Early Bird" [17]

In case of violation of at least one of the conditions (7), (8) and (9), the chaos is creeping in the orbits of daily and they become unstable.

3. CONCLUSION

The limits of evaluations of elusive hidden-perturbations of the gravitational constant in the Solar System; of the orbits of the daily satellites; in the Solar System to the center of the body of the nearest object; V satellite of Jupiter were received from the analysis of the amount of small perturbations. They are as follows: the gravitational constant of the Solar System:

$$10^{-13} \le \varepsilon(\Delta\dot{G}/G) \le 10^{-11}\,year^{-1}; \qquad (I)$$

Iovocentric coordinates of Jupiter's V satellite:

$$\begin{aligned} 0'',22 &\le \varepsilon[\Delta\xi\,(0-C)] \le 0''81, \\ -0'',61 &\le \varepsilon[\Delta\eta\,(0-C)] \le 0''54; \end{aligned} \qquad (II)$$

The drift motions of major semi-axes of daily satellites for the points where they intersect with the equator vary within

$$-4{,}2\,km/day \le \varepsilon(\Delta\dot{\rho}) \le 4{,}2\,km/day \qquad (III)$$

longitudes of daily satellites vary within:

$$-0{,}0053\deg./day \le \varepsilon(\Delta\dot{\lambda}) \le 0{,}0045\deg./day \qquad (IV)$$

The rotation period of the daily satellites vary within

$$-0{,}053 \le \varepsilon(\Delta \mathrm{T}) \le 0{,}055\ day \qquad (V)$$

Ratings (I) - (V) obtained from the analysis of the sum of infinitesimal perturbations of degree less than or equal to the errors of observations.

When satisfying to the boundary estimates (I) - (V), the Solar System, the orbits of V satellite and daily satellites are stable. Once these conditions are violated, the chaos is creeping in the Solar System, the orbits of V satellite and daily satellites and they become unstable.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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