Asian Research Journal of Mathematics

Volume 19, Issue 10, Page 57-66, 2023; Article no.ARJOM.104143
ISSN: 2456-477X

Fuzzy Fixed Point Theorems in Normal Cone Metric Space

P. Senthil Kumar [a] and P. Thiruveni [a*]

[a] Department of Mathematics, Rajah Serfoji Government College (Autonomous), (Affiliated To Bharathidasan University), Thanjavur, India.

Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

Article Information

DOI: 10.9734/ARJOM/2023/v19i10728

Open Peer Review History:
This journal follows the Advanced Open Peer Review policy. Identity of the Reviewers, Editor(s) and additional Reviewers, peer review comments, different versions of the manuscript, comments of the editors, etc are available here: https://www.sdiarticle5.com/review-history/104143

Original Research Article

Received: 02/06/2023
Accepted: 05/08/2023
Published: 12/08/2023

Abstract

In this paper, we proved a few fuzzy fixed point theorems in whole regular cone metric spaces, which can be the generalization of a few current consequences within side the literature.

Keywords: Normal cone; cone metric space; fixed point; fuzzy.

2020 AMS subject classifications: 54H25, 46S40, 47H10.

1 Introduction

Many researchers make the research under the fixed point theorems [1-3]. There exist some of generalizations of metric spaces, and one in all them is the cone metric spaces [4]. The notation of cone metric space is initiated via way of means of Huang and Zhang [5] and additionally they mentioned a few homes of the convergence of

---

**Corresponding author: Email: smile.thiruveni@gmail.com;*

sequences and proved the fuzzy fixed point theorems of a contraction mappings cone metric spaces [6]. Many authors have studied the life and forte of strict fuzzy constant factors for single valued mappings and multi valued mappings in metric spaces [7-10]. In this paper speak life and precise fixed point factor in entire ordinary cone metric spaces, which might be the generalization of a few current contraction principle.

Definition 1.1:

A subset S of E is called a cone if and only if :

1. S is closed, nonempty and $S \neq 0$
2. $ax + by \in S$ for all $x, y \in S$ and nonnegative real numbers a, b
3. $S \cap S^{-} = \{0\}$.

Given a cone $S \subset E$, we define a partial ordering $\leq$ with respect to S by $x \leq y$ if and only if $y - x \in P$. We will write $x < y$ that $x \leq y$ but $x \neq y$, while x, y will stand for $y - x \in int\, S$, where $int\, S$ denotes the interior of S. The cone P is called normal if there is a number $L > 0$ such that $0 \leq x \leq y$ implies $\|x\| \leq L\|y\|$ for all $x, y \in E$. The least positive number satisfying the above is called the normal constant. The cone L is called regular if every increasing sequence which is bounded from above is convergent. That is, if $\{x_n\}$ is sequence such that $x_1 \leq x_2 \leq \cdots \leq x_n \ldots \leq y$ for some $y \in E$, then there is $x \in E$ such that $\|x_n - x\| \to 0\ as\ n \to 0$.

Equivalently the cone S is regular if and only if every decreasing sequence which is bounded from below is convergent. It is well known that a regular cone is a normal cone. Suppose E is a Banach space, S is a cone in E with $int\, S \neq 0\ and \leq$ is partial ordering with respect to S.

Example 1.1:

Let $L > 1$ be given. Consider the real vector space with

$$E = \left\{ax + b : a, b\, \epsilon \mathbb{R}; x \in \left[1 - \frac{1}{k}, 1\right]\right\}$$

With supremum norm and the cone $S = \{ax + b : a \geq 0, b \geq 0\}$ in E. the cone S is ordinary and so normal.

Definition 1.2:

Suppose that E is real Banach space, then S is a cone in E with $int\, S \neq \emptyset$, and $\leq$ is partial ordering with respect to S. Let $\mathbb{X}$ be a nonempty set, a function $d: \mathbb{X} \times \mathbb{X} \to E$ is called a fuzzy cone metric on $\mathbb{X}$ if it satisfies the following conditions with

1. $d(x, y) \geq 0$, and $d(x, y) = 0$ if and only if $x = y\ \forall x, y \in X$,
2. $d(x, y) = d(y, x), \forall\, x, y \in X$,
3. $d(x, y) \leq d(x, z) + d(z, y), \forall\, x, y \in X$,

Then $(\mathbb{X}, d)$ is called a cone metric space ($\mathbb{C}_F\mathbb{M}$).

Definition 1.3:

A fuzzy cone metric space is a 3-tuple ($\mathbb{X}, \mathbb{C}_F\mathbb{M}, *$) such that S is a cone of E, $\mathbb{X}$ is nonempty set, $*$ is a continuous t-norm and M is a fuzzy set on $\mathbb{X} \times \mathbb{X} \times int(S)$ satisfying the following conditions, for all $x, y, z \in X$ and $t, s \in int(P)$ (that is $t \gg \Theta, s \gg \Theta$) .

1. $\mathbb{C}_F\mathbb{M}(x, y, t) > 0$,
2. $\mathbb{C}_F\mathbb{M}(x, y, t) = 1$ if and only if $x = y$,
3. $\mathbb{C}_F\mathbb{M}(x, y, t) = \mathbb{C}_F\mathbb{M}(y, x, t)$,
4. $\mathbb{C}_F\mathbb{M}(x, y, t) * \mathbb{C}_F\mathbb{M}(y, z, s) \leq \mathbb{C}_F\mathbb{M}(x, z, t + s)$,
5. $\mathbb{C}_F\mathbb{M}(x, y, .): int(P) \to [0,1]$ is continuous.

If $(\mathbb{X}, \mathbb{C}_F\mathbb{M}, *)$ is a fuzzy cone metric space, we will say that M is a fuzzy cone metric on $\mathbb{X}$.

Definition 1.4:

Let$(\mathbb{X}, \mathbb{C}_F\mathbb{M}, *)$ be a fuzzy cone metric space, $x \in \mathbb{X}$ and $\{x_n\}$ be a sequence in $\mathbb{X}$. Then $\{x_\mathfrak{n}\}$ is said to converge to x if for any $t \gg \Theta$ and any $\mathfrak{r} \in (0,1)$ there exists a natural number $\mathfrak{n}_0$ such that $\mathscr{M}(x_\mathfrak{n};\ x;\ t) > 1 - r$ for all $\mathfrak{n} \geq \mathfrak{n}_0$. We denote this by

$$\lim \mathfrak{n} \longrightarrow \infty\, x_\mathfrak{n} = x \text{ or } x_\mathfrak{n} \longrightarrow x \text{ as } \mathfrak{n} \to \infty.$$

Let $(\mathbb{X}, \mathbb{C}_F\mathbb{M}, *)$ be a fuzzy cone metric space, $x \in \mathbb{X}$ and $\{x_\mathfrak{n}\}$ be a sequence in $\mathbb{X}$. $\{x_\mathfrak{n}\}$ converges to x if and only if $\mathscr{M}(x_\mathfrak{n};\ x;\ t) \longrightarrow 1$ as $\mathfrak{n} \longrightarrow \infty$, for each $\mathfrak{t} \gg \Theta$.

Let $(\mathbb{X}, \mathbb{C}_F\mathbb{M}, *)$ be a fuzzy cone metric space and $\{x_\mathfrak{n}\}$ be a sequence in $\mathbb{X}$.

Then $\{x_\mathfrak{n}\}$ is said to be a Cauchy sequence if for any $0 < \varepsilon < 1$ and any $\mathfrak{t} \gg \Theta$.

There exists a natural number $\mathfrak{n}_0$ such that $\mathscr{M}(x_\mathfrak{n};\ x_\mathfrak{m};\ t) > 1 - \varepsilon$ for all $\mathfrak{n}, \mathfrak{m} \geq \mathfrak{n}_0$.

2 Main Result

Theorem 2.1:

Let $(\mathbb{X}, \mathbb{C}_F\mathbb{M}, *)$ be a complete fuzzy cone metric space and S be a normal cone with normal constant L. suppose the mapping $T: \mathbb{X} \times \mathbb{X} \times (0, \infty) \to [0, \infty)$ satisfies the following conditions:

$$\mathbb{C}_F\mathbb{M}(T_x, T_y, t) \leq \left(\frac{\mathbb{C}_F\mathbb{M}(x, T_x, t) + \mathbb{C}_F\mathbb{M}(y, T_y, t)}{\mathbb{C}_F\mathbb{M}(x, T_x, t) + \mathbb{C}_F\mathbb{M}(y, T_y, t) + l}\right)\mathbb{C}_F\mathbb{M}(x, y, t) \tag{1}$$

For all $x, y \in \mathbb{X}, where\ l \geq 1\ \&\ t \in \mathbb{X}. then$

i. T has fuzzy unique fixed point in $\mathbb{X}$.
ii. $T^n x'$ converges to a fuzzy fixed point, for all $x' \in \mathbb{X}$.

Proof :

i. Let $x_0 \in \mathbb{X}$ be arbitrary and choose a sequence $\{x_n\}$ such that $x_{n+1} = Tx_n$.

$$\mathbb{C}_F\mathbb{M}(x_{n+1,}x_n, t) = \mathbb{C}_F\mathbb{M}(Tx_n, Tx_{n-1}, t)$$
$$\leq \left(\frac{\mathbb{C}_F\mathbb{M}(x_n, Tx_n, t) + \mathbb{C}_F\mathbb{M}(x_{n-1}, Tx_{n-1}, t)}{\mathbb{C}_F\mathbb{M}(x_n, Tx_n, t) + \mathbb{C}_F\mathbb{M}(x_{n-1}, Tx_{n-1}, t) + l}\right)\mathbb{C}_F\mathbb{M}(x_n, x_{n-1,}t)$$
$$\leq \left(\frac{\mathbb{C}_F\mathbb{M}(x_{n,}x_{n+1}, t) + \mathbb{C}_F\mathbb{M}(x_{n-1,}x_n, t)}{\mathbb{C}_F\mathbb{M}(x_{n,}x_{n+1}, t) + \mathbb{C}_F\mathbb{M}(x_{n-1}, x_n, t) + l}\right)\mathbb{C}_F\mathbb{M}(x_{n,}x_{n-1}, t)$$

Take

$$\lambda_n = \frac{\mathbb{C}_F\mathbb{M}(x_{n,}x_{n+1,}t) + \mathbb{C}_F\mathbb{M}(x_{n-1,}x_{n,}t)}{\mathbb{C}_F\mathbb{M}(x_{n,}x_{n+1,}t) + \mathbb{C}_F\mathbb{M}d(x_{n-1,}x_{n,}t) + l},$$

We have

$$\mathbb{C}_F\mathbb{M}(x_{n+1}, x_n, t) \leq \lambda_n \mathbb{C}_F\mathbb{M}(x_n, x_{n-1}, t)$$
$$\leq (\lambda_n\lambda_{n-1})\, \mathbb{C}_F\mathbb{M}(x_{n-1}, x_{n-2}, t)$$
$$\leq (\lambda_n\lambda_{n-1} \dots \lambda_1)\, \mathbb{C}_F\mathbb{M}(x_1, x_0, t).$$

Observe that (λ_n) is non increasing, with positive terms. So,$\lambda_1 \ldots \lambda_n \leq {\lambda_1}^n$ and ${\lambda_1}^n \to 0$.

It follows that

$$\lim_{n\to\infty}(\lambda_1\lambda_2 \ldots \lambda_n) = 0.$$

Thus, it is verified that

$$\lim_{n\to\infty} \mathbb{C}_F\mathbb{M}(x_{n+1,}x_n, t) = 0$$

Now for all $m, n \in \mathbb{N}$ *and* $m > n$ *we have*

$$\begin{aligned}
&\mathbb{C}_F\mathbb{M}(x_{m,}x_n, t) \leq \mathbb{C}_F\mathbb{M}(x_n, x_{n+1}, t) + \mathbb{C}_F\mathbb{M}(x_{n+1}, x_{n+2}, t) + \cdots \mathbb{C}_F\mathbb{M}(x_{m-1}, x_m, t)\\
&\leq [(\lambda_n\lambda_{n-1} \ldots \lambda_1) + (\lambda_{n+1}\lambda_n \ldots \lambda_1) + \cdots + (\lambda_{m-1}\lambda_{m-2} \ldots \lambda_1)]\ \mathbb{C}_F\mathbb{M}(x_1, x_0, t)\\
&= \textstyle\sum_{k=n}^{m-1}(\lambda_k\lambda_{k-1} \ldots \lambda_1)\ \mathbb{C}_F\mathbb{M}(x_1, x_0, t)
\end{aligned}$$

$$\begin{aligned}
&\|\mathbb{C}_F\mathbb{M}(x_m, x_n, t)\| \leq L\|\textstyle\sum_{k=n}^{m-1}(\lambda_k\lambda_{k-1} \ldots \lambda_1)\ \mathbb{C}_F\mathbb{M}(x_1, x_0, t)\|\\
&\|\mathbb{C}_F\mathbb{M}(x_m, x_n, t)\| \leq L\textstyle\sum_{k=n}^{m-1}(\lambda_k\lambda_{k-1} \ldots \lambda_1)\ \|\ \mathbb{C}_F\mathbb{M}(x_1, x_0, t)\|\\
&\|\mathbb{C}_F\mathbb{M}(x_m, x_n, t)\| \quad \leq L\textstyle\sum_{k=n}^{m-1} a_k\ \|\mathbb{C}_F\mathbb{M}(x_1, x_0, t)\|,
\end{aligned}$$

Where $a_{k=(\lambda_k\lambda_{k-1}\ldots\lambda_1)}$ and L is normal constant of S.

Now $\lim_{k\to\infty}\frac{a_{k+1}}{a_k} < 1$ *and* $\sum_{k=1}^{\infty} a_k$is finite,

and $\sum_{k=n}^{m-1}(\lambda_k\lambda_{k-1} \ldots \lambda_1) \to 0, as\ m, n \to \infty$.

Hence $\{a_k\}$ is convergent by D' Alembert's ratio test, therefore $\{x_n\}$ is a cauchy sequence. There is $x' \in \mathbb{X}$ such that $x_n \to x'\ as\ n \to \infty$.

$$\begin{aligned}
&\mathbb{C}_F\mathbb{M}(Tx', x', t) \leq \mathbb{C}_F\mathbb{M}(Tx', Tx_n, t) + \mathbb{C}_F\mathbb{M}(Tx_n, x', t)\\
&\leq \left(\frac{\mathbb{C}_F\mathbb{M}(x',Tx',t)+\mathbb{C}_F\mathbb{M}(x_n,Tx_n,t)}{\mathbb{C}_F\mathbb{M}(x',Tx',t)+\mathbb{C}_F\mathbb{M}(x_n,Tx_n,t)+l}\right)\mathbb{C}_F\mathbb{M}(x_n, x', t) + \mathbb{C}_F\mathbb{M}(Tx_n, x', t)\\
&\leq \left(\frac{\mathbb{C}_F\mathbb{M}(x',Tx',t)+\mathbb{C}_F\mathbb{M}(x_n,Tx_{n+1},t)}{\mathbb{C}_F\mathbb{M}(x',Tx',t)+\mathbb{C}_F\mathbb{M}(x_n,Tx_{n+1},t)+l}\right)\mathbb{C}_F\mathbb{M}(x_n, x', t) + \mathbb{C}_F\mathbb{M}(Tx_{n+1}, x', t)\\
&\mathbb{C}_F\mathbb{M}(Tx', x', t) \leq 0\ \ as\ n \to \infty
\end{aligned}$$

Therefore $\|\mathbb{C}_F\mathbb{M}(Tx', x', t)\| = 0$.

Thus, $Tx' = x'$.

Uniqueness

Suppose x' and y' are two fixed points of T.

$$\begin{aligned}
&\mathbb{C}_F\mathbb{M}(x', y', t) = \mathbb{C}_F\mathbb{M}(Tx', Ty', t)\\
&\leq \left(\frac{\mathbb{C}_F\mathbb{M}(x',Tx',t)+\mathbb{C}_F\mathbb{M}(y',Ty',t)}{\mathbb{C}_F\mathbb{M}(x',Tx',t)+\mathbb{C}_F\mathbb{M}(y^1,Ty',t)+l}\right)\mathbb{C}_F\mathbb{M}(x', y', t)\\
&\leq 0
\end{aligned}$$

Therefore $\|\mathbb{C}_F\mathbb{M}(x', y', t)\| = 0$. Thus $x' = y'$.

Hence x' is an unique fuzzy fixed point of T.

ii. Now

$$\mathbb{C}_F\mathbb{M}(T^n x', x', t) = \mathbb{C}_F\mathbb{M}(T^{n-1}Tx', x', t) = \mathbb{C}_F\mathbb{M}(T^{n-1}x', x', t) = \mathbb{C}_F\mathbb{M}(T^{n-2}(Tx'), x', t) \dots = \mathbb{C}_F\mathbb{M}(Tx', Tx', t) = 0$$

Hence $T^n x'$ converges to a fuzzy fixed point, for all $x' \in \mathbb{X}$.

Corollary 2.1:

Let $(\mathbb{X}, \mathbb{C}_F\mathbb{M}, *)$ be a complete cone fuzzy metric space and S be a normal cone with normal constant L. suppose the mapping $T: \mathbb{X} \to \mathbb{X}$ satisfies the following conditions:

$$\mathbb{C}_F\mathbb{M}(T_x, T_y, t) \le \left(\frac{\mathbb{C}_F\mathbb{M}(x, T_x, t) + \mathbb{C}_F\mathbb{M}(y, T_y, t)}{\mathbb{C}_F\mathbb{M}(x, T_x, t) + \mathbb{C}_F\mathbb{M}(y, T_y, t) + 1}\right) \mathbb{C}_F\mathbb{M}(x, y, t) \tag{2}$$

For all $x, y \in \mathbb{X}$. Then

1. T has fuzzy unique fixed point in $\mathbb{X}$.
2. $T^n x'$ Converges to a fuzzy fixed point, for all $x' \in \mathbb{X}$.

Proof :

The proof of the corollary immediate by

Taking $l = 1$ in the above theorem.

Theorem 2.2:

Let $(\mathbb{X}, \mathbb{C}_F\mathbb{M}, *)$ be a complete fuzzy metric space and let T be a mapping from $\mathbb{X}$ into itself. Suppose that T satisfies the following condition:

$$\mathbb{C}_F\mathbb{M}(T_x, T_y, t) \le \left(\frac{\mathbb{C}_F\mathbb{M}(y, T_y, t)}{\mathbb{C}_F\mathbb{M}(x, T_x, t) + \mathbb{C}_F\mathbb{M}(y, T_y, t) + l}\right) \mathbb{C}_F\mathbb{M}(x, y, t) \tag{3}$$

For all $x, y \in \mathbb{X}, \textbf{where } l \ge 1$ & $t \in \mathbb{X}$. Then

1. T has unique fuzzy fixed point in $\mathbb{X}$.
2. $T^n x'$ Converges to a fuzzy fixed point, for all $x' \in \mathbb{X}$.

Proof :

1. Let $x_0 \in \mathbb{X}$ be arbitrary and choose a sequence $\{x_n\}$ such that $x_{n+1} = T\,x_n$

We have

$$\begin{aligned}
&\mathbb{C}_F\mathbb{M}(x_{n+1}, x_n, t) = \mathbb{C}_F\mathbb{M}(T\,x_n, T\,x_{n-1}, t) \\
&\le \left(\frac{\mathbb{C}_F\mathbb{M}(x_{n-1}, Tx_{n-1}, t)}{\mathbb{C}_F\mathbb{M}(x_n, x_{n+1}, t) + \mathbb{C}_F\mathbb{M}(x_{n-1}, x_n, t) + l}\right) \mathbb{C}_F\mathbb{M}(x_n, x_{n-1}, t) \\
&\le \left(\frac{\mathbb{C}_F\mathbb{M}(x_{n-1}, x_n, t)}{\mathbb{C}_F\mathbb{M}(x_n, x_{n+1}, t) + \mathbb{C}_F\mathbb{M}(x_{n-1}, x_n, t) + l}\right) \mathbb{C}_F\mathbb{M}(x_n, x_{n-1}, t) \\
&\le \left(\frac{\mathbb{C}_F\mathbb{M}(x_{n-1}, x_n, t)}{\mathbb{C}_F\mathbb{M}(x_n, x_{n+1}, t) + \mathbb{C}_F\mathbb{M}(x_{n-1}, x_n, t) + l}\right) \mathbb{C}_F\mathbb{M}(x_n, x_{n-1}, t)
\end{aligned}$$

Take

$$\lambda_n = \frac{\mathbb{C}_F\mathbb{M}(x_n, x_{n+1}, t) + \mathbb{C}_F\mathbb{M}(x_{n-1}, x_n, t)}{\mathbb{C}_F\mathbb{M}(x_n, x_{n+1}, t) + \mathbb{C}_F\mathbb{M}(x_{n-1}, x_n, t) + l}$$

We have

$$\mathbb{C}_F\mathbb{M}(x_{n+1}, x_n, t) \leq \lambda_n \mathbb{C}_F\mathbb{M}(x_n, x_{n-1}, t)$$
$$\leq (\lambda_n\lambda_{n-1})\mathbb{C}_F\mathbb{M}(x_{n-1}, x_{n-2}, t)$$
$$\leq (\lambda_n\lambda_{n-1} \dots \lambda_1)\, \mathbb{C}_F\mathbb{M}(x_1, x_0, t).$$

Observe that $\{\lambda_n\}$ is non- increasing, with positive terms.

So, $(\lambda_1 \dots \lambda_n) \leq {\lambda_1}^n \to 0$. It follows that

$$\lim_{n\to\infty}(\lambda_1\lambda_2 \dots \lambda_n) = 0.$$

Thus, it is verified that

$$\lim_{n\to\infty} \mathbb{C}_F\mathbb{M}\big(x_{n+1,}x_n, t\big) = 0.$$

Now for all $m, n \,\epsilon\, \mathbb{N}$ we have

$$\mathbb{C}_F\mathbb{M}\big(x_{m,}x_n, t\big) \leq \mathbb{C}_F\mathbb{M}(x_n, x_{n+1}, t) + \mathbb{C}_F\mathbb{M}(x_{n+1}, x_{n+2}, t) + \cdots \mathbb{C}_F\mathbb{M}(x_{m-1}, x_m, t)$$
$$\leq [(\lambda_n\lambda_{n-1} \dots \lambda_1) + (\lambda_{n+1}\lambda_n \dots \lambda_1) + \cdots + (\lambda_{m-1}\lambda_{m-2} \dots \lambda_1)]\, \mathbb{C}_F\mathbb{M}(x_1, x_0, t)$$
$$= \sum_{k=n}^{m-1}(\lambda_k\lambda_{k-1} \dots \lambda_1)\, \mathbb{C}_F\mathbb{M}(x_1, x_0, t)$$

$$\|\mathbb{C}_F\mathbb{M}(x_m, x_n, t)\| \leq L\left\|\sum_{k=n}^{m-1}(\lambda_k\lambda_{k-1} \dots \lambda_1)\, \mathbb{C}_F\mathbb{M}(x_1, x_0, t)\right\|$$
$$\|\mathbb{C}_F\mathbb{M}(x_m, x_n, t)\| \leq L\sum_{k=n}^{m-1}(\lambda_k\lambda_{k-1} \dots \lambda_1)\, \|\mathbb{C}_F\mathbb{M}(x_1, x_0, t)\|$$
$$\|\mathbb{C}_F\mathbb{M}(x_m, x_n, t)\| \quad \leq L\sum_{k=n}^{m-1} a_k\, \|\mathbb{C}_F\mathbb{M}(x_1, x_0, t)\|,$$

Where $a_{k=(\lambda_k\lambda_{k-1}\dots\lambda_1)}$ and L is normal constant of S.

Now $\lim_{k\to\infty}\frac{a_{k+1}}{a_k} < 1 \text{ and } \sum_{k=1}^{\infty} a_k$ is finite, and

$$\sum_{k=n}^{m-1} (\lambda_k\lambda_{k-1} \dots \lambda_1) \to 0, \text{as } m, n \to \infty.$$

Hence $\{a_k\}$ is convergent by D' Alembert's ratio test, therefore $\{x_n\}$ is a cauchy sequence. There is $x' \in \mathbb{X}$ such that $x_n \to x'$

$$\mathbb{C}_F\mathbb{M}(Tx', x', t) \leq \mathbb{C}_F\mathbb{M}(Tx', Tx_n, t) + \mathbb{C}_F\mathbb{M}(Tx_n, x', t)$$
$$\leq \left(\frac{\mathbb{C}_F\mathbb{M}(x', Tx', t) + \mathbb{C}_F\mathbb{M}(x_n, Tx_n, t)}{\mathbb{C}_F\mathbb{M}(x', Tx', t) + \mathbb{C}_F\mathbb{M}(x_n, Tx_n, t) + l}\right)\mathbb{C}_F\mathbb{M}(x_n, x', t) + \mathbb{C}_F\mathbb{M}(Tx_n, x', t)$$
$$\leq \left(\frac{\mathbb{C}_F\mathbb{M}(x', Tx', t) + \mathbb{C}_F\mathbb{M}(x_n, Tx_{n+1}, t)}{\mathbb{C}_F\mathbb{M}(x', Tx', t) + \mathbb{C}_F\mathbb{M}(x_n, Tx_{n+1}, t) + l}\right)\mathbb{C}_F\mathbb{M}(x_n, x', t) + \mathbb{C}_F\mathbb{M}(Tx_{n+1}, x', t)$$

$$\mathbb{C}_F\mathbb{M}(Tx', x', t) \leq 0 \quad \text{as } n \to \infty$$

Therefore $\left\|\mathbb{C}_F\mathbb{M}(Tx', x', t)\right\| = 0$. Thus, $Tx' = x'$.

Uniqueness

Suppose x' and y' are two fuzzy fixed points of T.

$$\mathbb{C}_F\mathbb{M}(x', y', t) = \mathbb{C}_F\mathbb{M}(Tx', Ty', t)$$
$$\leq \left(\frac{\mathbb{C}_F\mathbb{M}(x', Tx', t) + \mathbb{C}_F\mathbb{M}(y', Ty', t)}{\mathbb{C}_F\mathbb{M}(x', Tx', t) + \mathbb{C}_F\mathbb{M}(y', Ty', t) + l}\right)\mathbb{C}_F\mathbb{M}(x', y', t)$$
$$\leq 0$$

Therefore $\left\|\mathbb{C}_F\mathbb{M}(x', y', t)\right\| = 0.$ Thus $x' = y'$.

Hence $\boldsymbol{x}'$ is an unique fuzzy fixed point of T.

2. Now

$$\mathbb{C}_{\boldsymbol{F}}\mathbb{M}(\boldsymbol{T}^{\boldsymbol{n}}\boldsymbol{x}', \boldsymbol{x}', \boldsymbol{t}) = \mathbb{C}_{\boldsymbol{F}}\mathbb{M}(\boldsymbol{T}^{\boldsymbol{n-1}}\boldsymbol{T}', \boldsymbol{x}', \boldsymbol{t}) = \mathbb{C}_{\boldsymbol{F}}\mathbb{M}(\boldsymbol{T}^{\boldsymbol{n-1}}\boldsymbol{x}', \boldsymbol{x}', \boldsymbol{t}) = \mathbb{C}_{\boldsymbol{F}}\mathbb{M}(\boldsymbol{T}^{\boldsymbol{n-2}}(\boldsymbol{T}\boldsymbol{x}'), \boldsymbol{x}', \boldsymbol{t}) \ldots = \mathbb{C}_{\boldsymbol{F}}\mathbb{M}(\boldsymbol{T}\boldsymbol{x}', \boldsymbol{T}\boldsymbol{x}', \boldsymbol{t}) = \boldsymbol{0}$$

Hence $\boldsymbol{T}^{\boldsymbol{n}}\boldsymbol{x}'$ converges to a fuzzy fixed point, for all $\boldsymbol{x}' \in \mathbb{X}$.

Corollary 2.2:

Let $(\mathbb{X}, \mathbb{C}_{\boldsymbol{F}}\mathbb{M}, *)$ be a complete fuzzy metric space and let T be a mapping from $\mathbb{X}$ into itself. Suppose that T satisfies the following condition:

$$\mathbb{C}_{\boldsymbol{F}}\mathbb{M}(\boldsymbol{T}_{\boldsymbol{x}}, \boldsymbol{T}_{\boldsymbol{y}}, \boldsymbol{t}) \leq \left(\frac{\mathbb{C}_{\boldsymbol{F}}\mathbb{M}(\boldsymbol{y}, \boldsymbol{T}_{\boldsymbol{y}}, \boldsymbol{t})}{\mathbb{C}_{\boldsymbol{F}}\mathbb{M}(\boldsymbol{x}, \boldsymbol{T}_{\boldsymbol{x}}, \boldsymbol{t}) + \mathbb{C}_{\boldsymbol{F}}\mathbb{M}(\boldsymbol{y}, \boldsymbol{T}_{\boldsymbol{y}}, \boldsymbol{t}) + \boldsymbol{l}}\right) \mathbb{C}_{\boldsymbol{F}}\mathbb{M}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{t}) \tag{4}$$

For all $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{X}$,where $\boldsymbol{l} \geq \boldsymbol{1}$ & $\boldsymbol{t} \in \mathbb{X}$. Then

1. T has Specific fuzzy fixed point in $\mathbb{X}$.
2. $\boldsymbol{T}^{\boldsymbol{n}}\boldsymbol{x}'$ converges to a fuzzy fixed point, for all $\boldsymbol{x}' \in \mathbb{X}$.

Proof :

The proof of the corollary immediate by

Taking $\boldsymbol{l} = \boldsymbol{1}$ in the above theorem.

Theorem 2.3:

Let $(\mathbb{X}, \mathbb{C}_{\boldsymbol{F}}\mathbb{M}, *)$ be a complete cone metric space and P be a normal cone with ordinary constant L. suppose the mapping $\boldsymbol{T}: \mathbb{X} \to \mathbb{X}$ satisfies the following conditions:

$$\mathbb{C}_{\boldsymbol{F}}\mathbb{M}(\boldsymbol{T}_{\boldsymbol{x}}, \boldsymbol{T}_{\boldsymbol{y}}, \boldsymbol{t}) \leq \left(\frac{\mathbb{C}_{\boldsymbol{F}}\mathbb{M}(\boldsymbol{x}, \boldsymbol{T}_{\boldsymbol{y}}, \boldsymbol{t}) + \mathbb{C}_{\boldsymbol{F}}\mathbb{M}(\boldsymbol{y}, \boldsymbol{T}_{\boldsymbol{x}}, \boldsymbol{t})}{\mathbb{C}_{\boldsymbol{F}}\mathbb{M}(\boldsymbol{x}, \boldsymbol{T}_{\boldsymbol{x}}, \boldsymbol{t}) + \mathbb{C}_{\boldsymbol{F}}\mathbb{M}(\boldsymbol{y}, \boldsymbol{T}_{\boldsymbol{y}}, \boldsymbol{t}) + \boldsymbol{l}}\right) \left(\mathbb{C}_{\boldsymbol{F}}\mathbb{M}(\boldsymbol{x}, \boldsymbol{T}_{\boldsymbol{y}}, \boldsymbol{t}) + \mathbb{C}_{\boldsymbol{F}}\mathbb{M}(\boldsymbol{y}, \boldsymbol{T}_{\boldsymbol{x}}, \boldsymbol{t})\right) \tag{5}$$

For all $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{X}$, $\boldsymbol{where}\ \boldsymbol{l} \geq 1$ & $\boldsymbol{t} \in \mathbb{X}$. Then

1. T has unique fuzzy fixed point in $\mathbb{X}$.
2. $\boldsymbol{T}^{\boldsymbol{n}}\boldsymbol{x}'$converges to a fuzzy fixed point, for all $\boldsymbol{x}' \in \mathbb{X}$.

Proof :

Let $\boldsymbol{x}_{\boldsymbol{0}} \in \mathbb{X}$ be arbitrary and choose a sequence $\{\boldsymbol{x}_{\boldsymbol{n}}\}$ such that $\boldsymbol{x}_{\boldsymbol{n+1}} = \boldsymbol{T}\boldsymbol{x}_{\boldsymbol{n}}$.

$$\begin{aligned}
&\mathbb{C}_{\boldsymbol{F}}\mathbb{M}(\boldsymbol{x}_{\boldsymbol{n}}, \boldsymbol{x}_{\boldsymbol{n+1}}, \boldsymbol{t}) = \mathbb{C}_{\boldsymbol{F}}\mathbb{M}(\boldsymbol{T}\boldsymbol{x}_{\boldsymbol{n}}, \boldsymbol{T}\ \boldsymbol{x}_{\boldsymbol{n-1}}, \boldsymbol{t}) \\
&\leq \left(\frac{\mathbb{C}_{\boldsymbol{F}}\mathbb{M}(\boldsymbol{x}_{\boldsymbol{n}}, \boldsymbol{T}\boldsymbol{x}_{\boldsymbol{n-1}}, \boldsymbol{t}) + \mathbb{C}_{\boldsymbol{F}}\mathbb{M}(\boldsymbol{x}_{\boldsymbol{n-1}}, \boldsymbol{T}\boldsymbol{x}_{\boldsymbol{n}}, \boldsymbol{t})}{\mathbb{C}_{\boldsymbol{F}}\mathbb{M}(\boldsymbol{x}_{\boldsymbol{n}}, \boldsymbol{T}\boldsymbol{x}_{\boldsymbol{n}}, \boldsymbol{t}) + \mathbb{C}_{\boldsymbol{F}}\mathbb{M}(\boldsymbol{x}_{\boldsymbol{n-1}}, \boldsymbol{T}\boldsymbol{x}_{\boldsymbol{n-1}}, \boldsymbol{t}) + \boldsymbol{l}}\right) (\mathbb{C}_{\boldsymbol{F}}\mathbb{M}(\boldsymbol{x}_{\boldsymbol{n}}, \boldsymbol{T}\boldsymbol{x}_{\boldsymbol{n}}, \boldsymbol{t}) + \mathbb{C}_{\boldsymbol{F}}\mathbb{M}(\boldsymbol{x}_{\boldsymbol{n-1}}, \boldsymbol{T}\boldsymbol{x}_{\boldsymbol{n-1}}, \boldsymbol{t})) \\
&\leq \left(\frac{\mathbb{C}_{\boldsymbol{F}}\mathbb{M}(\boldsymbol{x}_{\boldsymbol{n}}, \boldsymbol{x}_{\boldsymbol{n}}, \boldsymbol{t}) + \mathbb{C}_{\boldsymbol{F}}\mathbb{M}(\boldsymbol{x}_{\boldsymbol{n-1}}, \boldsymbol{x}_{\boldsymbol{n+1}}, \boldsymbol{t})}{\mathbb{C}_{\boldsymbol{F}}\mathbb{M}(\boldsymbol{x}_{\boldsymbol{n}}, \boldsymbol{x}_{\boldsymbol{n+1}}, \boldsymbol{t}) + \mathbb{C}_{\boldsymbol{F}}\mathbb{M}(\boldsymbol{x}_{\boldsymbol{n-1}}, \boldsymbol{x}_{\boldsymbol{n}}, \boldsymbol{t}) + \boldsymbol{l}}\right) \left(\mathbb{C}_{\boldsymbol{F}}\mathbb{M}(\boldsymbol{x}_{\boldsymbol{n}}, \boldsymbol{x}_{\boldsymbol{n+1}}, \boldsymbol{t}) + \mathbb{C}_{\boldsymbol{F}}\mathbb{M}(\boldsymbol{x}_{\boldsymbol{n}}, \boldsymbol{x}_{\boldsymbol{n-1}}, \boldsymbol{t})\right) \\
&\leq \left(\frac{\mathbb{C}_{\boldsymbol{F}}\mathbb{M}(\boldsymbol{x}_{\boldsymbol{n-1}}, \boldsymbol{x}_{\boldsymbol{n+1}}, \boldsymbol{t})}{\mathbb{C}_{\boldsymbol{F}}\mathbb{M}(\boldsymbol{x}_{\boldsymbol{n}}, \boldsymbol{x}_{\boldsymbol{n+1}}, \boldsymbol{t}) + \mathbb{C}_{\boldsymbol{F}}\mathbb{M}(\boldsymbol{x}_{\boldsymbol{n-1}}, \boldsymbol{x}_{\boldsymbol{n}}, \boldsymbol{t}) + \boldsymbol{l}}\right) \left(\mathbb{C}_{\boldsymbol{F}}\mathbb{M}(\boldsymbol{x}_{\boldsymbol{n}}, \boldsymbol{x}_{\boldsymbol{n+1}}, \boldsymbol{t}) + \mathbb{C}_{\boldsymbol{F}}\mathbb{M}(\boldsymbol{x}_{\boldsymbol{n}}, \boldsymbol{x}_{\boldsymbol{n-1}}, \boldsymbol{t})\right) \\
&\leq \left(\frac{\mathbb{C}_{\boldsymbol{F}}\mathbb{M}(\boldsymbol{x}_{\boldsymbol{n-1}}, \boldsymbol{x}_{\boldsymbol{n}}, \boldsymbol{t}) + \mathbb{C}_{\boldsymbol{F}}\mathbb{M}(\boldsymbol{x}_{\boldsymbol{n}}, \boldsymbol{x}_{\boldsymbol{n+1}}, \boldsymbol{t})}{\mathbb{C}_{\boldsymbol{F}}\mathbb{M}(\boldsymbol{x}_{\boldsymbol{n}}, \boldsymbol{x}_{\boldsymbol{n+1}}, \boldsymbol{t}) + \mathbb{C}_{\boldsymbol{F}}\mathbb{M}(\boldsymbol{x}_{\boldsymbol{n-1}}, \boldsymbol{x}_{\boldsymbol{n}}, \boldsymbol{t}) + \boldsymbol{l}}\right) \left(\mathbb{C}_{\boldsymbol{F}}\mathbb{M}(\boldsymbol{x}_{\boldsymbol{n}}, \boldsymbol{x}_{\boldsymbol{n+1}}, \boldsymbol{t}) + \mathbb{C}_{\boldsymbol{F}}\mathbb{M}(\boldsymbol{x}_{\boldsymbol{n}}, \boldsymbol{x}_{\boldsymbol{n-1}}, \boldsymbol{t})\right)
\end{aligned}$$

Take

$$\boldsymbol{\lambda}_{\boldsymbol{n}} = \frac{\mathbb{C}_{F}\mathbb{M}(\boldsymbol{x}_{\boldsymbol{n-1}}, \boldsymbol{x}_{\boldsymbol{n}}, \boldsymbol{t}) + \mathbb{C}_{F}\mathbb{M}(\boldsymbol{x}_{\boldsymbol{n}}, \boldsymbol{x}_{\boldsymbol{n+1}}, \boldsymbol{t})}{\mathbb{C}_{\boldsymbol{F}}\mathbb{M}(\boldsymbol{x}_{\boldsymbol{n}}, \boldsymbol{x}_{\boldsymbol{n+1}}, \boldsymbol{t}) + \mathbb{C}_{\boldsymbol{F}}\mathbb{M}(\boldsymbol{x}_{\boldsymbol{n-1}}, \boldsymbol{x}_{\boldsymbol{n}}, \boldsymbol{t}) + \boldsymbol{l}}$$

We have

$$\mathbb{C}_F\mathbb{M}(x_{n+1,}x_n,t) \le \lambda_n\left(\mathbb{C}_F\mathbb{M}(x_{n,}x_{n+1},t) + \mathbb{C}_F\mathbb{M}(x_{n,}x_{n-1},t)\right)$$
$$(1-\lambda_n)\,\mathbb{C}_F\mathbb{M}(x_{n+1,}x_n,t) \le \lambda_n\,\mathbb{C}_F\mathbb{M}(x_{n,}x_{n-1},t)$$
$$\mathbb{C}_F\mathbb{M}(x_{n+1,}x_n,t) \le \frac{\lambda_n}{(1-\lambda_n)}\mathbb{C}_F\mathbb{M}(x_{n,}x_{n-1},t)$$

$$\le \frac{\lambda_n\lambda_{n-1}}{(1-\lambda_n)(1-\lambda_{n-1})}\,\mathbb{C}_F\mathbb{M}(x_{n-1,}x_{n-2},t)$$
$$\le \frac{\lambda_n\lambda_{n-1}\dots\lambda_1}{(1-\lambda_n)(1-\lambda_{n-1})\dots(1-\lambda_1)}\mathbb{C}_F\mathbb{M}(x_{1,}x_0,t).$$
$$\le \gamma_n\,\mathbb{C}_F\mathbb{M}(x_{1,}x_0,t)$$

Where

$$\gamma_n = \frac{\lambda_n\lambda_{n-1}\dots\lambda_1}{(1-\lambda_n)(1-\lambda_{n-1})\dots(1-\lambda_1)}$$

Observe that $\{\lambda_n\}$ is non increasing, with positive terms. So, $(\lambda_1 \dots \lambda_n) \le {\lambda_1}^n \to 0$.

It follows that

$$\lim_{n\to\infty}(\lambda_1\lambda_2\dots\lambda_n) = 0.$$

Therefore

$$\lim_{n\to\infty}\gamma_n = 0$$

Thus, it is verified that

$$\lim_{n\to\infty}\mathbb{C}_F\mathbb{M}(x_{n+1,}x_n,t) = 0.$$

Now for all $m, n\,\epsilon\mathbb{N}$ we have

$$\mathbb{C}_F\mathbb{M}(x_{m,}x_n,t) \le \mathbb{C}_F\mathbb{M}(x_n,x_{n+1},t) + \mathbb{C}_F\mathbb{M}(x_{n+1},x_{n+2},t) + \cdots \mathbb{C}_F\mathbb{M}(x_{m-1},x_m,t)$$
$$\le [(\gamma_n + \gamma_{n+1} + \cdots + \gamma_{m-1})]\,\mathbb{C}_F\mathbb{M}(x_1,x_0,t)$$
$$\le \sum_{k=n}^{m-1}\gamma_k\,\mathbb{C}_F\mathbb{M}(x_1,x_0,t)$$

$$\|\mathbb{C}_F\mathbb{M}(x_m,x_n,t)\| \le L\left\|\sum_{k=n}^{m-1}\gamma_k\,\mathbb{C}_F\mathbb{M}(x_1,x_0,t)\right\|$$
$$\|\mathbb{C}_F\mathbb{M}(x_m,x_n,t)\| \le L\sum_{k=n}^{m-1}\gamma_k\,\|\mathbb{C}_F\mathbb{M}(x_1,x_0,t)\|$$
$$\|\mathbb{C}_F\mathbb{M}(x_m,x_n,t)\| \quad \le L\sum_{k=n}^{m-1}a_k\,\|\mathbb{C}_F\mathbb{M}(x_1,x_0,t)\|,$$

where $a_{k=}\gamma_k$ and L is normal constant of S.

Now $\lim_{k\to\infty}\frac{a_{k+1}}{a_k} < 0$ *and* $\sum_{k=1}^{\infty}a_k$ is finite.

Since $\sum_{k=n}^{m-1}\gamma_k$ is convergent by D' Alembert's ratio test as $m \to \infty$.

Therefore $\{x_n\}$ is a cauchy sequence.

There is $x' \in \mathbb{X}$ such that $x_n \to x'$ as $n \to \infty$

$$\mathbb{C}_F\mathbb{M}(Tx',x',t) \le \mathbb{C}_F\mathbb{M}(Tx',Tx_n,t) + \mathbb{C}_F\mathbb{M}(Tx_n,x',t)$$
$$\le \left(\frac{\mathbb{C}_F\mathbb{M}(x',Tx_n,t)+\mathbb{C}_F\mathbb{M}(x_n,Tx',t)}{\mathbb{C}_F\mathbb{M}(x',Tx_n)+\mathbb{C}_F\mathbb{M}(x_{n},,Tx',t)+l}\right)\mathbb{C}_F\mathbb{M}(x_n,x',t) + \mathbb{C}_F\mathbb{M}(Tx_n,x',t)$$

$$\leq \left(\frac{\mathbb{C}_F\mathbb{M}(x',x_{n+1},t)+\mathbb{C}_F\mathbb{M}(x_n,Tx',t)}{\mathbb{C}_F\mathbb{M}(x',x_{n+1},t)+\mathbb{C}_F\mathbb{M}(x_n,Tx',t)+l}\right)\mathbb{C}_F\mathbb{M}(x_n,x',t)+\ \mathbb{C}_F\mathbb{M}(Tx_{n+1},x',t)$$
$$\mathbb{C}_F\mathbb{M}(Tx',x',t)\leq 0 \quad \text{as } n\to\infty$$

Therefore $\left\|\mathbb{C}_F\mathbb{M}(x',Tx',t)\right\| = 0$.

Thus, $Tx' = x'$.

Uniqueness

Suppose x' and y' are two fuzzy fixed points of T.

$$\mathbb{C}_F\mathbb{M}(x',y',t) = \mathbb{C}_F\mathbb{M}(Tx',Ty',t)$$
$$\leq \left(\frac{\mathbb{C}_F\mathbb{M}(x',Ty',t)+\mathbb{C}_F\mathbb{M}(y',Tx',t)}{\mathbb{C}_F\mathbb{M}(x',Tx',t)+\mathbb{C}_F\mathbb{M}(y',Ty',t)+l}\right)\left(\mathbb{C}_F\mathbb{M}(x',Tx',t)+\mathbb{C}_F\mathbb{M}(y',Ty',t)\right)$$
$$\leq 0$$

Therefore $\left\|\mathbb{C}_F\mathbb{M}(x',y',t)\right\| = 0.$ Thus $x' = y'$.

Hence x' is an unique fuzzy fixed point of T.

(ii) Now

$$\mathbb{C}_F\mathbb{M}(T^n x',x',t) = \mathbb{C}_F\mathbb{M}(T^{n-1}(Tx'),x',t) = \mathbb{C}_F\mathbb{M}(T^{n-1}x',x',t) = \mathbb{C}_F\mathbb{M}(T^{n-2}(Tx'),x',t)\ldots = \mathbb{C}_F\mathbb{M}(Tx',x',t) = 0$$

Hence $T^n x'$ converges to a fuzzy fixed point, for all $x' \in \mathbb{X}$.

Corollary 2.3:

Let $(\mathbb{X},\mathbb{C}_F\mathbb{M},*)$ be a complete fuzzy metric space and let T be a mapping from S be a normal cone with normal constant L. Suppose the mapping $T:\mathbb{X}\to\mathbb{X}$ Satisfies the subsequent condition:

$$\mathbb{C}_F\mathbb{M}(T_x,T_y,t) \leq \left(\frac{\mathbb{C}_F\mathbb{M}(x,T_y,t)+\mathbb{C}_F\mathbb{M}(y,T_x,t)}{\mathbb{C}_F\mathbb{M}(x,T_x,t)+\mathbb{C}_F\mathbb{M}(y,T_y,t)+l}\right)\left(\mathbb{C}_F\mathbb{M}(x,T_x,t)+\mathbb{C}_F\mathbb{M}(y,T_y,t)\right) \quad (6)$$

For all $x,y \in \mathbb{X}$. Then

1. T has unique fuzzy fixed point in $\mathbb{X}$.
2. $T^n x'$ converges to a fuzzy fixed point, for all $x' \in \mathbb{X}$.

Proof :

The evidence of the corollary on the spot by taking $L = 1$ within side the above theorem.

Competing Interests

Authors have declared that no competing interests exist.

References

[1] Senthil Kumar P, Thiruveni P. Estimation of fuzzy metric spaces from metric spaces, Ratio Mathematica. 2021;41:64-70.

[2] Senthil kumar P, Thiruveni P, Dhamodharan D. Some results on common fixed point in fuzzy 2-normed linear space, Advances And Applications In Mathematical Sciences. 2022;21(11):6425-6435.

[3] Senthil kumar P, Thiruveni P, Rostam K Saeed. Fuzzy Fixed Point Theorems for $\boldsymbol{\gamma}$ – WeakContractive Mappings in 3 Metric Spaces, Journal of Algebraic Statistics. 2022;13(2):723-729.
ISSN: 1309-3452.

[4] Krishnakumar R, Dhamodharan D. Fixed point theorems in normal cone metric space, International J. of Math. Sci. & Engg. Appls. (IJMSEA). 2016;10(III):213-224.
ISSN 0973-9424.

[5] Huang LG, Zhang. Cone metric spaces and fixed point theorems of contractive mappings, J. Math. Anal. 2007;332:1468-1476.

[6] Saif Ur Rehman, Hong-Xu Li. Fixed point theorems in fuzzy cone metric spaces; J. Non linear Sci. 2017;10:5763-5769.
ISSN: 2008-1898.

[7] George A, Veeramani P. On Some Results of Analysis For Fuzzy Metric Spaces, Fuzzy Sets and Systems. 1997;90:365-368.

[8] Karmosil O, Michalek J. Fuzzy metric and Statistical Metric Spaces, Kybernetica. 1975;11:326-334.

[9] Krishnakumar R, Mani T, Dhamodharan D. Fixed point (α, β, γ)- contraction Mapping under simulation functions in Banach Spaces, Malaya Journal of Matematik. 2021;9(1):745-750.

[10] Krishnakumar R, Dinesh K, Dhamodharan D, Some Fixed point (α, β)- weak contraction on Fuzzy Metric space, International Journal Sci. Res. In Mathematical and Statistical Science. 2018;5(3):1-6.

Peer-review history:
The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)
https://www.sdiarticle5.com/review-history/104143