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Design of Fractional Order Controllers Using Genetic Algorithms

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Short Research Article

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Abstract

This paper presents the development of fractional order $PI^{\lambda}D^{\mu}$ and fuzzy- controllers for control of dynamic plant. The fractional order derivative and integral are described. The designs of fractional order $PI^{\lambda}D^{\mu}$ - and fuzzy controllers have been done. The parameters of fractional order controllers are turned using a real coded Genetic Algorithm (GA). The performances of the proposed control systems are illustrated through application examples. Fractional calculus has shown improvement in time response characteristics of feedback control system through the use of non-integer order derivatives and integrals.

Keywords: Fuzzy controller, fractional calculus, feedback control.

1 Introduction

Nowadays fractional calculus is an effective tool to describe some characteristics of technological process and widely used in the field of automatic control. The performance of the control system is defined by producing the target response in minimal time. One of an efficient way to improve the performance of control system is the use of fractional calculus in the design of control system [1]. This is achieved using fractional derivatives and integrals instead of first order whole valued derivatives and integrals [2,3,4]. Number of researches have been devoted to the design of fractional-order controllers [5,6]. It was shown that the fractional order PID extends the capabilities of the classical counterpart and, thus, has a wider domain of application.

Uncertainty of environment, insufficiency and fuzziness of information needs the use of intelligent control system for control of dynamic plants. Design process of these control system needs to take into account   specific characters of technological processes. Fuzzy logic is one of effective means for information processing. The use of fuzzy logic in estimation of situations and creation of logic rules in control models simplifies decision making and information processing. In this paper the use of fractional calculus for the design of deterministic and fuzzy controllers are considered. During design of these controllers the main problem is the determining of the near optimal values of the controller's parameters.

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As known the design of these controllers needs a great knowledge about control object, turning experience of the controller's parameters. Traditionally the turning experience is basically done manually, by trail and error. This is tedious and some time takes more time for finding the proper values of the parameters of the controller. Number of algorithms have been designed to simplify the turning procedure. Nowadays different techniques are used for finding the parameters of fractional-order controllers.

This paper presents the design of deterministic- and fuzzy fractional order controllers using real coded GA. GA operators are applied to find the parameters of controllers. The comparative analyses of the fractional controllers with traditional ones have been given.

2 Fractional Calculus in Control

It was shown that the usage of fractional calculus in controller design allows improving the performance of the control system. The fractional calculus (FC) is a useful tool to describe technological process, such as flow, heat, electricity, mechanics or fluid dynamics [1,6]. Nowadays FC approach is used in many areas of science and engineering, Fractional calculus (FC) is a representations of integration and differentiation to a non-integer order and is denoted by ${}_aD_t^\alpha$ fundamental operator, where a and t are the limits of the operation, α is a fractional order which may be complex number [1-4]. ${}_aD_t^\alpha$ is defined as follows

$$ {}_aD_t^\alpha = \begin{cases} \dfrac{d^\alpha}{dt^\alpha} & , R(\alpha) > 0 \\ 1 & , R(\alpha) = 0 \\ \displaystyle\int_a^t (d\tau)^{-\alpha} & , R(\alpha) < 0 \end{cases} \tag{1} $$

There are two set of definitions for general fractional differentiation and integration. Commonly used definitions are the Riemann Liouville (RL), Caputto and the Grünwald–Letnikov (GL) definitions [7,8]. The most used definition is the RL definition given by:

$$ {}_aD_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_a^t (t-\tau)^{n-\alpha-1} f(\tau) d\tau \tag{2} $$

For $(n-1 < \alpha < n)$ and $\Gamma(x)$ is the well known Euler's Gamma function. The Gamma function is defined as

$$ \Gamma(x) = \int e^{-t} t^{x-1} dt, \quad \mathrm{x} > 0 \tag{3} $$

in special case when x=n

$$\Gamma(n) = (n-1)(n-2)...(2)(1) = (n-1)!$$
$$\Gamma(n+1) = n! = n\Gamma(n) \tag{4}$$

L. E. Euler considered fractional differentiation for the function x^p, when he extended the formula $\frac{d^n x^p}{dx^n} = p(p-1)(p-2)...(p-n+1)x^{p-n} = \frac{p!}{(p-n)!} z^{p-n}$ to $n = \alpha$, where α is arbitrary

$$D_x^{\alpha} x^p = \frac{\Gamma(p+1)}{\Gamma(p-\alpha+1)} x^{p-\alpha} \tag{5}$$

This formula has great interest in this paper. Another definition of fractional differintegral introduced by Caputo [8]. Caputo's definition can be written as:

$${}_aD_t^{\alpha} f(t) = \frac{1}{\Gamma(\mathrm{n}-\alpha)} \int_a^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau \tag{6}$$

Other definition is formulated by Grunwald-Letnikov (GL). GL definition of the fractional order differ-integral is formulated as follows:

$${}_aD_t^{\alpha} f(t) = \lim_{h \to 0} \frac{1}{h^{\alpha}} \sum_{j=0}^{[(t-\alpha)/h]} (-1)^j \binom{\alpha}{j} f(t-jh) \tag{7}$$

where $\binom{\alpha}{j}$ is a flooring-operator determined as.

$$\binom{\alpha}{j} = \frac{\Gamma(\alpha+1)}{\Gamma(\alpha-j+1)} \tag{8}$$

The fractional integrals and derivatives can also be defined in the s-domain. Considering null initial conditions, they are given by the simple form:

$$D^{\alpha} f(t) = L^{-}\{s^{\alpha} F(s)\} \tag{9}$$

here $\alpha \in$ R, L represents the Laplace operator and $F(x) = L\{x(t)\}$.

3 Structure of Fractional Order Controller

3.1 Fractional order PID controller

Proportional, integral, derivative (PID) controller uses the effect of the error (the proportional part), the derivative of the error (the differential part) and the integral of the error (the integral part). PID controllers are widely used in industries. Using the formula of conventional PID controller, the fractional order $PI^{\lambda}D^{\mu}$ controller can be formulated as

$$u = K_p e(t) + K_d \frac{d^{\mu}}{dt^{\mu}} e(t) + K_i \int e^{\lambda}(t)dt, \quad or$$
$$u = K_p e(t) + K_d D^{\mu} e(t) + K_i I^{\lambda} e(t) \tag{10}$$

As shown in formula the proportionality constant K_p, acting on the error, the differential constant K_d acting on fractional derivative of error and the integral constant K_i acting on fractional integral of error. Using equation (5) and formula of (10) of fractional order $PI^{\lambda}D^{\mu}$ controller the control signal for object could be determined .The structure of fractional order $PI^{\lambda}D^{\mu}$ controller is given in Fig. 1.

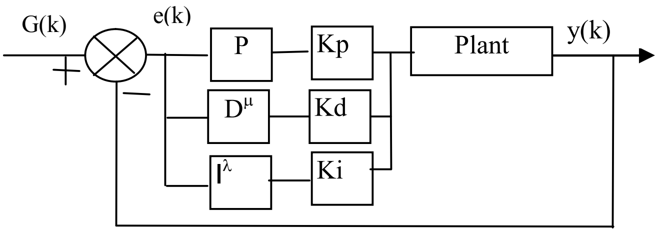

Fig. 1. Structure of $PI^{\lambda}D^{\mu}$ control system

The design of $PI^{\lambda}D^{\mu}$ control system includes the finding of the proper values of the parameters of Kp, Ki, Kd, λ and μ. In the paper GA is used for the finding proper values of the parameters.

3.2 Fuzzy fractional Order Controller

Fuzzy technology has elements and inference mechanism for describing uncertain systems. The design of fuzzy controllers is well studied in the literatures. The basic input-output relation in fuzzy controller consist in input variables error $e(k)$, change in error $\Delta e(k)$ and output control signal u(k). This relation is constructed using If-Then rules extracted from experts in control field. Linguistic expressions in antecedent and consequent parts of IF-THEN rules are describing the operator's actions that are converted into a fully-structured control algorithm. Here error $e(k)$ and change in error $\Delta e(k)$ correspond to the proportional and differential part of PID controller. In the paper integral part is additionally added to the fuzzy controller. The structure of the fractional order fuzzy controller used for control of dynamic plants is given in Fig. 2.

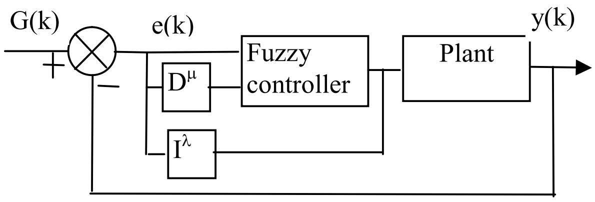

Fig. 2. Structure of fuzzy fractional order control system

Controller output depends on the value of error, fractional derivative and fractional integral of error. That is controller output is the function of error, change of error and integral value of error $u(k) = f(e(k), D^{\mu}e(k), I^{\lambda}(k))$. $D^{\mu}e(k)$ and $I^{\lambda}(k)$ are discrete values of fractional derivative and integral. Here $I^{\lambda} = D^{-\lambda}$.

The kernel of controller is fuzzy knowledge base that includes relation between error $e(k)$, change in error $\Delta e(k)$ and control actions u(k). This relation is given in Table 1. Knowledge base (KB) consists of production rules, which have IF...THEN... form. It is constructed on the base of experimental data and knowledge of experts and specialists.

Table 1. Knowledge base table

Force u		**Change-in-error e'**						
		NL	**NM**	**NS**	**Z**	**PS**	**PM**	**PL**
Error e	NL	NL	NL	NL	NM	NM	NS	Z
	NM	NL	NL	NM	NM	NS	Z	PS
	NS	NL	NM	NM	NS	Z	PS	PM
	Z	NL	NM	NS	Z	PS	PM	PL
	PS	NM	NS	Z	PS	PM	PL	PL
	PM	NS	Z	PS	PM	PL	PL	PL
	PL	Z	PS	PM	PL	PS	PM	PL

In Fig. 3 a fragment of knowledge base is given.

If e(k) is NL and Δe(k) is NS Then u(k) is NM
If e(k) is NM and Δe(k) is Z Then u(k) is NM
If e(k) is NS and Δe(k) is PM Then u(k) is PS
…
If e(k) is PB and Δe(k) is NS Then u(k) is PM

Fig. 3. Fragment of rule base

The Inference engine procedure is applied to find fuzzy controller output. The common output is determined in the result of summing rule base output and integral value of control action.

4 GA Learning

In this paper, the real coded genetic algorithm (GA) is applied for searching optimal values of the parameters of the controller. GA is a directed random search method that exploits historical information to direct the search into the region of better performance within the search space [9]. GA allows to solve local optimal problem that have traditional searching methods, and find a global optimal solution. The operation principle of GA is shown in Fig. 4. At first step for each parameter the set of initial values are generated randomly. Each solution that includes parameters values is called chromosome. Chromosomes consist of genes and represent the controller's parameters. The number of chromosome is defined by population size. GA evaluates each generation using evaluation function and determine the value of fitness function. If among these solutions there is solution that satisfies the given criteria then the optimal solution is found. Otherwise GA operators - selection, crossover and mutation are used to change variable values. The square of error (SER) criteria is taken to evaluate the solutions.

$$E = \sum_{k=1}^{n} e^2(k) = \sum_{k=1}^{n} (G(k) - y(k))^2 \tag{10}$$

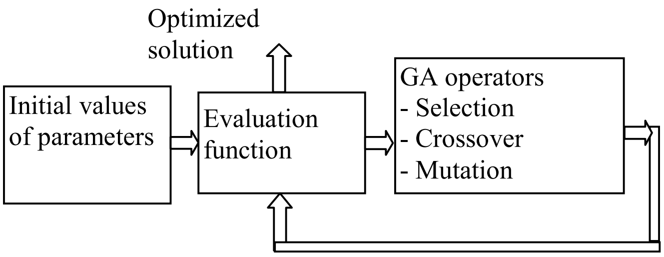

Fig. 4. GA operation scheme

Initially the chromosomes are generated randomly. GA operators are used to train the parameter values in chromosomes. The main GA operators are selection, crossover and mutation. The selection gives more chance to population members (or solutions) that have higher fitness. In the paper a tournament selection is applied for selection of new generation. In this selection mechanism, two population members are selected and their fitness values are compared. The member with high fitness is selected for the next generation.

After selection operator the crossover is applied to the selected two pairs in order to produce the next generation. Here the aim is to give children chance to differ from their parents, and hope that some of these children can be closer to the optimal destination than their parents. A crossover operation is applied for the updating of individuals and generating a new solution. The selections of individuals are performed according to crossover rate. The high value of the crossover rate led to a quick generation of a new solution. The typical value of the crossover rate is in the interval [0.5,1]. If we accept the individuals $X=(x_1, x_2, \ldots, x_n)$and $Y=(y_1, y_2, \ldots, y_n)$ as two parent members then after crossover operation, the new members will have the form $X' = (x'_1, x'_2, \ldots, x'_n)$ and $Y' = (y'_1, y'_2, \ldots, y'_n)$. The crossover operation is performed as

$$\begin{aligned} x_i^{'} &= x_i + \delta(y_i - x_i) \\ y_i^{'} &= x_i + \delta(x_i - y_i) \end{aligned} \tag{11}$$

when $F(X)>F(Y)$. Here x_i and y_i are the i-th genes of the parents X and Y, $x_i^{'}$ and $y_i^{'}$ are the i-th genes of the parents X' and Y'. The value δ is changed between 0 and 1.

After crossover the simple mutation operation is applied to the genes. A random number is generated for each gene. If the generated random number is less than the mutation rate, then the corresponding gene is selected for mutation. In this operation a small random number, taken from the interval [0,1], is added to the selected gene in order to determine its new value. A large value of the mutation rate leads to a purely random search. The typical value of mutation rate is lay in the interval [0,0.1].

5 Simulation

The performance of the fractional order controllers are investigated for control purpose. The designs of $PI^{\lambda}D^{\mu}$and fuzzy controllers have been performed according the structures given in Figs. 1 and 2, respectively. The proposed fractional order controllers are used for the control of the dynamic plant described by the following difference equation.

$$y(k) = \frac{y(k-1)y(k-2)(y(k-1)+2.5)}{(1+y(k-1)^2+y(k-2)^2)} + u(k) \tag{12}$$

At first simulation the design of fractional order $PI^{\lambda}D^{\mu}$controller has been performed using GA. In the simulation the PID controller was augmented with fractional calculus by setting the order of the error's integral and derivative to λ and μ respectively. The simulation has been performed using Matlab(R2012a) package on PC Pentium(R) (CPU2020@2.90Ghz, 2.90 GHz). Real coded GA, given in section 4, is applied to train the parameters of the controller. Population size was taken 50, crossover rate was 0.6, and mutation rate was 0.08. Initially a set of solutions (a population), each of which describes a parameters of controller, is generated randomly. Then the updates of the parameters are performed using selection, crossover and mutation operators. After each parameter update operations SER performance of control system is computed using formula (10). The parameter updates and SER computation have been done for each generation. The parameter values corresponding to the least value of SER are selected as best values of parameters. In the result of GA training the appropriate values of K_p, K_i, K_d, λ and μ are determined as 0.2592, 0.99, 0.2725, 0.9, 0.4 respectively. For these parameters the value of square error was obtained as E=23.5672. The GA learning is continued for 1000 iterations. Fig. 5 depicts the values of SER obtained in the result of GA training. The controller uses fractional integrals and derivatives computed by the equation of (5), while the PID difference methods to compute first order integrals and derivatives.

Fig. 6 depicts time response characteristic of fractional order of $PI^{\lambda}D^{\mu}$control system. The simulation results of $PI^{\lambda}D^{\mu}$ controller are compared with the results of standard integer order PID control system. For this reason above described GA training procedure is applied to determine the values of the coefficients of integer order PID controller. In result of simulation the corresponding values of the controller coefficients K_p, K_i and K_d were determined as 0.3576, 0, 0.5095 respectively. For these values the sum of square errors of the PID control system was obtained as 31.515022. The obtained results are averaged over ten independent runs. The simulation results demonstrate that fractional order control system has better (less) SER performance than integer order control system.

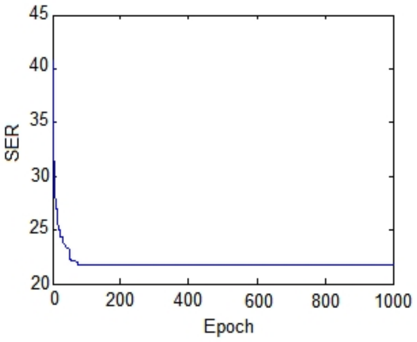

Fig. 5. GA training results

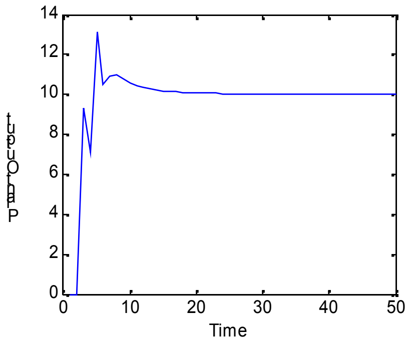

Fig. 6. Time response characteristic

In the next stage the design of fractional fuzzy control system is performed for the plant of (12). The fuzzy controller was augmented with fractional calculus by setting the order of the error's integral and derivative to λ and μ respectively. In the result of GA training the values of λ and μ coefficients are determined as 0.8 and 0.2 respectively. The square error for fractional fuzzy control system was obtained as E=47.86754. For comparative analysis the simulation of control system using integer order fuzzy controller has been done. The SER of integer order fuzzy control system was obtained as E=49.16147. Fig. 7 depicts the transient response characteristic of fractional and integer order fuzzy control systems. In figure curve line is the response characteristic of integer order, dashed line is the response characteristic of fractional order fuzzy control system. Comparing SER performances and response characteristics of fractional order- and integer order fuzzy control systems it becomes clear that the fractional order controllers produces better results than the integer ones.

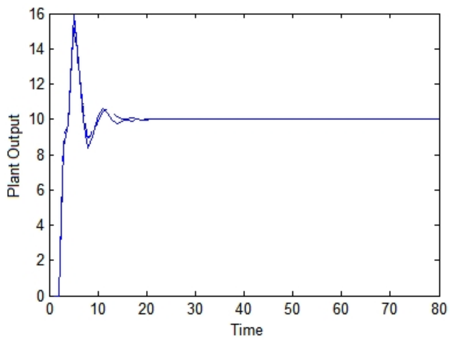

Fig. 7. Time response characteristics of fuzzy control system: curve line is the response characteristic of integer order, dashed line- fractional order fuzzy control system

6 Conclusion

The paper describes the design of fractional order deterministic and fuzzy controllers using real coded GA. Euler formula is applied to find the values of fractional differential and fractional integral. The approach is used in computing the output control signal of fractional order $PI^{\mu}D^{\lambda}$ and fuzzy controllers. The structures of fractional order controllers are presented. GA operators are applied to find the parameters of $PI^{\mu}D^{\lambda}$ and fuzzy controllers. The designed controllers are tested on the dynamic plants. Obtained simulation results demonstrate that the performances of fractional order controllers are better than their integer analogs. The future work will consider the real life applications of fractional order controllers.

Competing Interests

Author has declared that no competing interests exist.

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