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Derivatives Involving I-Function of Two Variables and General Class of Polynomials

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Abstract

This paper presents some derivative formulas of I-function of two variables involving general class of polynomials. The special cases of our derivatives yield interesting results.

Keywords: I-function; Mellin-Barnes contour integral; general class of polynomials.

1 Introduction

The well known H-function of one variable defined by Fox [1] and proved the H-function as a symmetric Fourier kernel to Meijers's G-function [2]. The H-function is often called Fox's H-function. Later on many researchers studied and developed H-function. In 1997, Rathie [3] introduced a new function in the literature namely the I-function which is useful in Mathematics, Physics and other branches of applied mathematics. In 2012, Shantha et al. [4] defined and studied I-function of two variables and in 2013, Shantha et al. [5] evaluated some differentiation formulas for I-function of two variables. In the present paper we establish derivative formulae of I-function of two variables involving general class of polynomials.

We shall utilize the following formulae and notations in the present investigation. The I-function of two variables defined by Shantha et al. [4] (and also see [6]) in following manner.

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(1.1) $I[z_1, z_2]$

$$= I^{0,n_1:m_2,n_2;m_3,n_3}_{p_1,q_1:p_2,q_2;p_3,q_3}\left[\begin{matrix} z_1 \\ z_2 \end{matrix}\middle| \begin{matrix} (a_j;\alpha_j,A_j;\xi_j)_{1,p_1} : (c_j,C_j;U_j)_{1,p_2};(e_j,E_j;P_j)_{1,p_3} \\ (b_j;\beta_j,B_j;\eta_j)_{1,q_1} : (d_j,D_j;V_j)_{1,q_2};(f_j,F_j;Q_j)_{1,q_3} \end{matrix}\right]$$

$$= \frac{1}{(2\pi i)^2}\int_{L_s}\int_{L_t} \phi(s,t)\theta_1(s)\theta_2(t)z_1^s z_2^t \, ds\, dt$$

Where

$$\phi(s,t) = \frac{\prod_{j=1}^{n_1}\Gamma^{\xi_j}\left(1-a_j+\alpha_j s+A_j t\right)}{\prod_{j=n_1+1}^{p_1}\Gamma^{\xi_j}\left(a_j-\alpha_j s-A_j t\right)\prod_{j=1}^{q_1}\Gamma^{\eta_j}\left(1-b_j+\beta_j s+B_j t\right)}$$

$$\theta_1(s) = \frac{\prod_{j=1}^{n_2}\Gamma^{U_j}(1-c_j+C_j s)\prod_{j=1}^{m_2}\Gamma^{V_j}(d_j-D_j s)}{\prod_{j=n_2+1}^{p_2}\Gamma^{U_j}(c_j-C_j s)\prod_{j=m_2+1}^{q_2}\Gamma^{V_j}(1-d_j+D_j s)}$$

$$\theta_2(t) = \frac{\prod_{j=1}^{n_3}\Gamma^{P_j}(1-e_j+E_j t)\prod_{j=1}^{m_3}\Gamma^{Q_j}(f_j-F_j t)}{\prod_{j=n_3+1}^{p_3}\Gamma^{P_j}(e_j-E_j t)\prod_{j=m_3+1}^{q_3}\Gamma^{Q_j}(1-f_j+F_j t)}$$

where $n_j, p_j, q_j (j = 1,2,3)$, $m_j (j = 2,3)$ are non negative integers such that $0 \le n_j \le p_j$, $q_1 \ge 0$, $0 \le m_j \le q_j$ $(j = 2,3)$ (not all zero simultaneously), α_j, A_j $(j = 1, \ldots, p_1)$; β_j, B_j $(j = 1, \ldots, q_1)$, $C_j (j = 1, \ldots, p_2)$, $D_j (j = 1, \ldots, q_2)$, $E_j (j = 1, \ldots, p_3)$, $F_j (j = 1, \ldots, q_3)$ are positive quantities, a_j $(j = 1, \ldots, p_1)$, $b_j (j = 1, \ldots, q_1)$, $c_j (j = 1, \ldots, p_2)$, $d_j (j = 1, \ldots, q_2)$, $e_j (j = 1, \ldots, p_3)$ and $f_j (j = 1, \ldots, q_3)$ are complex numbers. The exponents ξ_j, η_j, U_j, V_j, P_j, Q_j may take non integer values.

L_s and L_t are suitable contours of Mellin-Barnes type. More over, the contour L_s is in the complex s-plane and runs from $\sigma_1 - i\infty$ to $\sigma_1 + i\infty$ (σ_1 real), so that all the poles of $\Gamma^{V_j}(d_j - D_j s)$ $(j = 1, \ldots, m_2)$ lie to the right of L_s and all poles of $\Gamma^{U_j}(1-c_j+C_j s)$ $(j = 1, \ldots, n_2)$, $\Gamma^{\xi_j}\left(1-a_j+\alpha_j s+A_j t\right)$ $(j = 1, \ldots, n_1)$ lie to the left of L_s. Similar conditions for L_t follows in complex t-plane. The detailed conditions of this function can be found in Shantha et al. [4].

The class of polynomials [7] (and also see [8])

$$(1.2)\quad S_n^m[x] = \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k} x^k \text{ , n = 0, 1, 2, . . .}$$

Where m is an arbitrary positive integer and the coefficients $A_{n,k}$ $(n,\ k \geq 0)$ are arbitrary constants. And also used the following notations.

$$(1.3)\quad D_x = \frac{d}{dx}$$

$$(1.4)\quad D_x^r f(x) = \frac{d^r}{dx^r} f(x)$$

$$(1.5)\quad (xD_x)^r f(x) = \left(x\frac{d}{dx} \right)^r f(x)$$

$$(1.6)\quad (D_x x)^r f(x) = \left(\frac{d}{dx} x \right)^r f(x).$$

2 Main Results

In this section we derive the following theorems.

Theorem 1. Prove that

$$(2.1)\quad D_x^r \left\{ S_n^m[ax^\lambda] I[z_1 x^{h_1}, z_2 x^{h_2}] \right\}$$

$$= \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k} a^k x^{\lambda k - r}$$

$$I^{0, n_1+1 : m_2, n_2 ; m_3, n_3}_{p_1, q_1+1 : p_2, q_2 ; p_3, q_3} \left[\begin{matrix} z_1 x^{h_1} \\ z_2 x^{h_2} \end{matrix} \middle| \begin{matrix} (-\lambda k; h_1, h_2; 1), (a_j; \alpha_j, A_j; \xi_j)_{1,p_1} : \\ (b_j; \beta_j, B_j; \eta_j)_{1,q_1}, (r - \lambda k; h_1, h_2; 1) : \end{matrix} \right.$$

$$\left. \begin{matrix} (c_j, C_j; U_j)_{1,p_2} ; (e_j, E_j; P_j)_{1,p_3} \\ (d_j, D_j; V_j)_{1,q_2} ; (f_j, F_j; Q_j)_{1,q_3} \end{matrix} \right]$$

Where λ complex number and h_1, h_2 are real and positive.

Proof. To prove this theorem, we consider

$$D_x^r \left\{ S_n^m[ax^\lambda] I[z_1 x^{h_1}, z_2 x^{h_2}] \right\}$$

And express I-function of two variables as contour integral (1.1), the general class of polynomials as series (1.2) and evaluating the derivative with help of the notation (1.4), we get

(2.4) $D_x^r\left\{S_n^m[ax^\lambda]I[z_1x^{h_1},z_2x^{h_2}]\right\}$

$$= \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k}\, a^k \frac{1}{(2\pi i)^2} \int_{L_s}\int_{L_t} \Big\{ \phi(s,t)\,\theta_1(s)\,\theta_2(t) z_1^s z_2^t \times \prod_{j=0}^{r-1}\left(\lambda k + h_1 s + h_2 t - j\right) x^{\lambda k + h_1 s + h_2 t - r} \Big\} ds\, dt$$

using the expression

$$\text{(2.5)} \quad \prod_{j=0}^{r-1}\left(\lambda k + h_1 s + h_2 t - j\right) = \frac{\Gamma(1+\lambda k + h_1 s + h_2 t)}{\Gamma(1+\lambda k + h_1 s + h_2 t - r)}$$

in (2.4) and simplifying with the help of (1.1), we obtain the result (2.1).

Theorem 2. Prove that

(2.6) $(xD_x - k_1)(xD_x - k_2).......(xD_x - k_r)\left\{S_n^m[ax^\lambda]I[z_1x^{h_1},z_2x^{h_2}]\right\}$

$$= \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k} \left(ax^{\lambda}\right)^k I^{0,n_1+r:m_2,n_2;m_3,n_3}_{p_1,q_1+r:p_2,q_2;p_3,q_3}\left[\begin{matrix} z_1x^{h_1} \\ z_2x^{h_2}\end{matrix}\middle| \begin{matrix} (k_j-\lambda k;h_1,h_2;1)_{1,r},(a_j;\alpha_j,A_j;\xi_j)_{1,p_1} : (c_j,C_j;U_j)_{1,p_2};(e_j,E_j;P_j)_{1,p_3} \\ (b_j;\beta_j,B_j;\eta_j)_{1,q_1},(1+k_j-\lambda k;h_1,h_2;1)_{1,r} : (d_j,D_j;V_j)_{1,q_2};(f_j,F_j;Q_j)_{1,q_3} \end{matrix}\right]$$

Where λ, k_j are complex numbers and h_1, h_2 are real and positive.

Proof. To prove this theorem, we consider

$$(xD_x - k_1)(xD_x - k_2)......(xD_x - k_r)\left\{S_n^m[ax^\lambda]I[z_1x^{h_1},z_2x^{h_2}]\right\}$$

Express I-function of two variables with the contour integral (1.1), the general class of polynomials as series (1.2) and evaluating the derivatives with help of the notation (1.5), we have

(2.7) $(xD_x - k_1)(xD_x - k_2).....xD_x - k_r)\left\{S_n^m[ax^\lambda]I[z_1x^{h_1},z_2x^{h_2}]\right\}$

$$= \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k}\, a^k \frac{1}{(2\pi i)^2} \int_{L_s}\int_{L_t} \Big\{ \phi(s,t)\,\theta_1(s)\,\theta_2(t) z_1^s z_2^t \times \prod_{j=1}^{r}\left(\lambda k - k_j + h_1 s + h_2 t\right) x^{\lambda k + h_1 s + h_2 t} \Big\} ds\, dt$$

By using

(2.8) $\prod_{j=1}^{r}\left(\lambda k-k_j+h_1 s+h_2 t\right)=\prod_{j=1}^{r}\frac{\Gamma\left(1+\lambda k-k_j+h_1 s+h_2 t\right)}{\Gamma\left(\lambda k-k_j+h_1 s+h_2 t\right)}$ in (2.7) and simplifying with the help of (1.1), we have the result (2.6).

Theorem 3. Prove that

$$(2.9)\quad (D_x x-k_1)(D_x x-k_2)....(D_x x-k_r)\left\{S_n^m[ax^{\lambda}]I[z_1x^{h_1},z_2x^{h_2}]\right\}$$
$$=\sum_{k=0}^{[n/m]}\frac{(-n)_{mk}}{k!}A_{n,k}\left(ax^{\lambda}\right)^k$$
$$I^{0,n_1+r:m_2,n_2;m_3,n_3}_{p_1,q_1+r:p_2,q_2;p_3,q_3}\left[\begin{matrix}z_1x^{h_1}\\ z_2x^{h_2}\end{matrix}\middle|\begin{matrix}(k_j-\lambda k-1;h_1,h_2;1)_{1,r},(a_j;\alpha_j,A_j;\xi_j)_{1,p_1}:\\ (b_j;\beta_j,B_j;\eta_j)_{1,q_1},(k_j-\lambda k;h_1,h_2;1)_{1,r}:\end{matrix}\right.$$
$$\left.\begin{matrix}(c_j,C_j;U_j)_{1,p_2};(e_j,E_j;P_j)_{1,p_3}\\ (d_j,D_j;V_j)_{1,q_2};(f_j,F_j;Q_j)_{1,q_3}\end{matrix}\right]$$

Where λ, k_j are complex numbers and h_1, h_2 are real and positive.

Proof. Proof of (2.9) is same as that of (2.1) and (2.6).

3 Special Cases

(i) By writing $k_1=k_2=\ldots\ldots=k_r=0$ in (2.6), we get

$$(3.1)\quad (xD_x)^r\left\{S_n^m[ax^{\lambda}]I[z_1x^{h_1},z_2x^{h_2}]\right\}$$
$$=\sum_{k=0}^{[n/m]}\frac{(-n)_{mk}}{k!}A_{n,k}\left(ax^{\lambda}\right)^k$$
$$I^{0,n_1+r:m_2,n_2;m_3,n_3}_{p_1,q_1+r:p_2,q_2;p_3,q_3}\left[\begin{matrix}z_1x^{h_1}\\ z_2x^{h_2}\end{matrix}\middle|\begin{matrix}(-\lambda k;h_1,h_2;1)_{1,r},(a_j;\alpha_j,A_j;\xi_j)_{1,p_1}:\\ (b_j;\beta_j,B_j;\eta_j)_{1,q_1},(1-\lambda k;h_1,h_2;1)_{1,r}:\end{matrix}\right.$$
$$\left.\begin{matrix}(c_j,C_j;U_j)_{1,p_2};(e_j,E_j;P_j)_{1,p_3}\\ (d_j,D_j;V_j)_{1,q_2};(f_j,F_j;Q_j)_{1,q_3}\end{matrix}\right]$$

Where λ is complex number and h_1, h_2 are real and positive.

(ii) when $k_1=k_2=\ldots..=k_r=0$ in (2.9), we get

$$(3.2)\quad (D_x x)^r \left\{ S_n^m[ax^\lambda] I[z_1 x^{h_1}, z_2 x^{h_2}] \right\}$$
$$= \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k} \left(ax^\lambda\right)^k$$
$$I^{0,n_1+r:m_2,n_2;m_3,n_3}_{p_1,q_1+r:p_2,q_2;p_3,q_3}\left[\begin{matrix} z_1 x^{h_1} \\ z_2 x^{h_2} \end{matrix}\middle| \begin{matrix} (-\lambda k-1;h_1,h_2;1)_{1,r},(a_j;\alpha_j,A_j;\xi_j)_{1,p_1}: \\ (b_j;\beta_j,B_j;\eta_j)_{1,q_1},(-\lambda k;h_1,h_2;1)_{1,r}: \end{matrix}\right.$$
$$\left.\begin{matrix} (c_j,C_j;U_j)_{1,p_2};(e_j,E_j;P_j)_{1,p_3} \\ (d_j,D_j;V_j)_{1,q_2};(f_j,F_j;Q_j)_{1,q_3} \end{matrix}\right]$$

Where λ is complex number and h_1, h_2 are real and positive.

(iii) Taking $\lambda = 0$, $a = 1$ in (2.1),(2.6) and (2.9), we obtain three derivative formulae established by Shantha et al. ((3.1), (3.2), (3.3) of [5]).

(iv) If $\lambda = 0$, $a = 1$, $p_1=q_1=n_1=0$, and $z_2\to 0$ in (2.1), (2.6) and (2.9), gives corresponding results involving I-function established by Vyas and Rathie [9].

(v) By using $\xi_j = \eta_j = U_j = V_j = P_j = Q_j = 1$ in (2.1), (2.6) and (2.9), then we get derivative formulae involving H-function of two variables and general class of polynomials.

(vi) If we take $\lambda = 0$, $a = 1$, $\xi_j = \eta_j = U_j = V_j = P_j = Q_j = 1$, $p_1=q_1=n_1=0$ and $z_2\to 0$ in (2.1), (2.6) and (2.9), we obtain differentiation formulae for H-function established by Gupta et al. [10] and Nair [11].

It may be of interest to conclude that our Theorems 1, 2 and 3 have more applications than what we have indicated here rather briefly.

4 Conclusion

Thus the generalized derivatives of product of general class of polynomials and I-function of two variables transformed as I-function of two variables but expression involving more terms. Also one can find same formulae involving general class of polynomial, I-function of r-variables.

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Competing Interests

Authors have declared that no competing interests exist.

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