



## Stagnation Point Flow of a Williamson Fluid over a Nonlinearly Stretching Sheet with Thermal Radiation

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### Authors' contributions

*This work was carried out in collaboration between all authors. Author MM designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript.*

*Authors MM and CKK managed the analyses of the study. Author JS managed the literature searches. All authors read and approved the final manuscript.*

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### ABSTRACT

The present analysis deals with the study of stagnation point flow of a Williamson fluid over a nonlinearly stretching sheet with thermal radiation. The partial differential equations governing this phenomenon were transformed into coupled nonlinear ordinary differential equations with suitable similarity transformations. These equations were then solved by numerical technique known as Keller Box method. The various parameters such as Prandtl number (Pr), velocity ratio parameter ( $\varepsilon$ ), Williamson parameter ( $\lambda$ ) and Radiation parameter (R) and non linear stretching parameter (n) determining the velocity and temperature distributions, the local Skin friction coefficient and the local Nusselt number governing such a flow were also analyzed. On analysis it was found that the Williamson fluid parameter ( $\lambda$ ) decreased both the fluid velocity whereas an increase in ( $\lambda$ ) increased wall skin-friction coefficient. The wall temperature gradient increased with an increase in Pr but decreased with radiation parameter R.

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## 1. INTRODUCTION

The study of two-dimensional boundary layer flow and heat mass transfer over a nonlinear stretching surface has enormous applications in different areas. Its industrial applications include aerodynamic extrusion of plastic sheets, condensation process of metallic plate in a cooling bath, extrusion of a polymer sheet from a dye or electronic chips and many others. During the manufacture of these sheets, the final product of desired characteristics depends on the rate of cooling and the process of stretching. Crane [1] demonstrated an exact analytical solution for the steady two-dimensional flow due to a stretching surface in a quiescent fluid. His work was later extended by many authors wherein they considered various flow aspects and obtained similarity solutions [2-7].

All the above mentioned studies confined their discussions by assuming stretching sheet to be linear. However, study by Gupta and Gupta [8] revealed that the stretching of the sheet may not necessarily be linear. Vajravelu [9] studied the flow and heat transfer characteristics in a viscous fluid over a nonlinearly stretching sheet without heat dissipation effect. Cortell [10] has worked on viscous flow and heat transfer over a nonlinearly stretching sheet. Raptis and Perdakis [11] studied viscous flow over a nonlinear stretching sheet in the presence of a chemical reaction and magnetic field. The comprehensive references regarding the flows over a non linear stretching under different physical situations can be obtained in the literature [12-17].

On the other hand, Hiemenz [18] first studied the steady flow in the neighbourhood of a stagnation-point. Chiam [19] investigated the problem by combining the works of Hiemenz [18] and Crane [1] and he assumed that the plate is stretched in its own plane with a velocity proportional to the distance from the stagnation point such that this velocity is identical to the stagnation flow velocity in the inviscid free stream. He then concluded that the flow near the plate is identical to the inviscid flow far from the plate and hence found no boundary layer structure near the plate. Mahapatra and Gupta [20] reinvestigated the stagnation-point flow problem towards a stretching sheet with different

stretching and straining rates and found two kinds of boundary layer near the sheet depending on the ratio of the stretching and straining rates.

In non-Newtonian fluids, the most commonly encountered fluids are pseudoplastic fluids whose behaviour has been explained by proposing different models like power law model, Carreau model, Cross model, Ellis model and Williamson fluid model. The Williamson model of non-Newtonian fluid is very much similar to the blood as it almost completely describes the behaviour of blood flow. Williamson [21] explained the flow of pseudoplastic materials and proposed a model equation to describe the flow of pseudoplastic fluids and the results were experimentally verified. The effects of heat and mass transfer peristaltic flow of Williamson fluid in a vertical annulus has been discussed by Nadeem et al. [22]. Vajravelu et al. [23] analyzed peristaltic transport of a Williamson fluid with permeable walls in asymmetric channel, Dapra and Scarpi [24] developed the perturbation solution for a Williamson fluid injected into a rock fracture. Cramer et al. [25] showed that the Williamson fluid model fits the experimental data of polymer solutions and particle suspensions better than other models. The power law model for pseudoplastic fluids predicts that with increasing shear rate, the apparent/effective viscosity should decrease indefinitely, which means infinite viscosity at rest and zero viscosity as the shear rate approaches infinity. A real fluid has both minimum and maximum effective viscosities depending upon the molecular structure of the fluid. Both the minimum ( $\mu_{\infty}$ ) and maximum viscosities ( $\mu_0$ ) are considered in the Williamson fluid model. So, for pseudoplastic fluids (for which the apparent viscosity does not go to zero at infinity), it will give better results. Several studies were carried out for Williamson fluid model under different flow patterns [26-30].

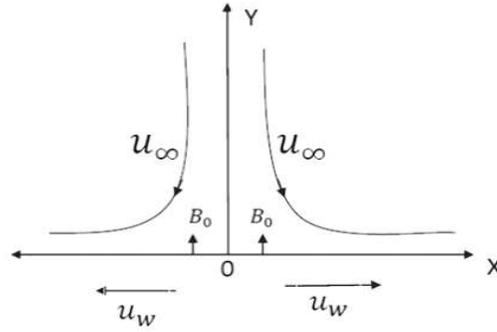
A new dimension introduced to the study of flow and heat transfer over a nonlinear stretching sheet is by considering the effect of thermal radiation. When the difference between the sheet and the ambient temperature is large the thermal radiation effects are quite significant, and also at high operating temperature the presence of thermal radiation alters the thermal boundary layer structure. Thermal radiation plays an

important role in controlling heat transfer process in polymer processing industry and also in the field of space technology. In such industrial processes knowledge of radiative heat transfer becomes relevant. Krishnamurthy et al. [31] studied effect of chemical reaction on MHD boundary layer flow and melting heat transfer of Williamson nanofluid in porous medium. Gorla et al [32] obtained dual solutions for stagnation-point flow and convective heat transfer of a Williamson nanofluid past a stretching/shrinking sheet. Thermally radiative three-dimensional flow of Jeffrey nanofluid with internal heat generation and magnetic field has been discussed by Shehzad et al. [33]. Suction, viscous dissipation and thermal radiation effects on the flow and heat transfer of a power-law fluid past an infinite porous plate was analyzed by Rafael Cortell [34]. Further information on radiative heat transfer flows can be found in [35-38].

However there are no studies on the stagnation point flow of a Williamson fluid over a nonlinearly stretching sheet with thermal radiation. Very few studies include such fluid model. To the best of our knowledge this is the first study to consider this fluid model over a nonlinear stretching sheet .The main objective of the present work is to study the stagnation point flow of a Williamson fluid over a non linear stretching sheet with thermal radiation. The momentum and energy equations are transformed into a set of ordinary differential systems and then solved implementing the Keller Box method.

**2. MATHEMATICAL FORMULATION**

Consider a steady two dimensional laminar flow of an incompressible Williamson fluid over a horizontally stretching sheet coinciding with the plane  $y = 0$  as depicted in the Fig1. Two equal and opposite forces are applied along the  $x$ -axis to produce stretching, i.e., the  $x$ -axis is taken along the stretching surface in the direction of the motion with the slot as the origin, and the  $y$ -axis is perpendicular to the sheet in the outward direction towards the fluid. The flow is assumed to be confined in a region  $y > 0$ . It is also assumed that the velocity of the ambient fluid is  $U_\infty(x) = ax^n$  where  $a$  is a positive constant and the stretching sheet velocity is  $U_w(x) = cx^n$  where  $c > 0$  is constant of proportionality,  $T_w, T_\infty$  are the uniform temperature at the sheet, free stream temperature respectively and  $n$  is the power index related to the surface stretching speed.



**Fig. 1. Physical sketch of the given problem**

Details of the Williamson fluid model can be found in Nadeem [39]. For the present fluid model, the Cauchy stress tensor  $S$  is defined as;

$$S = -pI + \tau$$

$$\tau = \left( \mu_\infty + \frac{\mu_0 - \mu_\infty}{1 - \Gamma\gamma} \right) A_1$$

where  $p$  is the pressure ,  $I$  is the identity vector,  $\tau$  is the extra stress tensor,  $\mu_0, \mu_\infty$  are the limiting viscosities at zero and infinite shear rates,  $\Gamma > 0$  is a time constant,  $A_1$  is the first Rivlin-Erickson tensor, and  $\gamma$  is defined as

$$\gamma = \sqrt{\frac{\pi}{2}}, \pi = trace(A_1^2)$$

where  $\pi$  is the second invariant strain tensor. Here we have only considered the case for which

$$\mu_\infty = 0 \text{ and } < 1 .$$

Then, we obtain  $\tau = \left( \frac{\mu_0}{1 - \Gamma\gamma} \right) A_1$  or  $\tau = \mu_0(1 + \Gamma\gamma)A_1$

The governing boundary layer equations for flow and heat transfer in the absence of body force are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + U_\infty \frac{\partial U_\infty}{\partial x} + \sqrt{2} \nu \Gamma \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\kappa}{\rho c_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} \tag{3}$$

The corresponding boundary conditions [9] are

$$\begin{aligned} u = U_w = cx^n, \quad v = 0, \quad T = T_w, \quad \text{at } y = 0 \\ u \rightarrow U_\infty, \quad T \rightarrow T_\infty \quad \text{at } y \rightarrow \infty \end{aligned} \quad (4)$$

where  $x$  and  $y$  denotes the cartesian coordinates along and normal to the sheet, respectively,  $u, v$  are the velocity components in  $x, y$  direction respectively. Here,  $c$  ( $c > 0$ ) is the surface stretching sheet related parameter,  $\nu$  is the kinematic viscosity,  $\rho$  is the viscosity,  $\kappa$  is the thermal diffusivity of the fluid,  $C_p$  is the specific heat.

Following Rosseland approximation, the radiative heat flux  $q_r$  is modelled as

$$q_r = -\left(\frac{4\sigma^*}{3k^*}\right) \frac{\partial T^4}{\partial y} \quad (5)$$

where  $\sigma^*$  is the Stefan-Boltzmann constant and  $k^*$  is the absorption coefficient. Assuming that the differences in temperature within the flow are such that  $T^4$  can be expressed as a linear combination of the temperature, we expand  $T^4$  in a Taylor's series about  $T_\infty$  we obtain

$$\frac{\partial q_r}{\partial y} = -\frac{16T_\infty^3 \sigma^*}{3k^*} \frac{\partial^2 T}{\partial y^2} \quad (6)$$

using eq.(8) in eq.(3) we obtain

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\kappa}{\rho C_p} \frac{\partial^2 T}{\partial y^2} + \frac{1}{\rho C_p} \frac{16T_\infty^3 \sigma^*}{3k^*} \frac{\partial^2 T}{\partial y^2} \quad (7)$$

By using the following suitable similarity transformations

$$\begin{aligned} u = cx^n f'(\eta), \quad v = \sqrt{c\nu} \frac{(n+1)}{2} x^{\frac{n-1}{2}} [f(\eta) \\ + \frac{n-1}{n+1} \eta f'(\eta)] \\ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \eta = y \sqrt{\frac{c(n+1)}{2\nu}} x^{\frac{n-1}{2}} \end{aligned} \quad (8)$$

The governing partial differential equations are transformed into following ordinary coupled nonlinear differential equations

$$f''' + ff'' - \frac{2n}{n+1} (f'^2 - \epsilon^2) + \lambda f'' f''' = 0 \quad (9)$$

$$\left(1 + \frac{4R}{3}\right) \theta'' + Pr f \theta' = 0 \quad (10)$$

and the boundary conditions are transformed into

$$f'(\eta) = 1, \quad f(\eta) = 0, \quad \theta(\eta) = 1 \quad \text{at } \eta = 0$$

$$f'(\eta) \rightarrow \epsilon, \quad \theta(\eta) \rightarrow 0 \quad \text{at } \eta = \infty \quad (11)$$

Where  $\epsilon = \frac{\mu C_p}{\kappa}$ ,  $\epsilon = \frac{a}{c}$ ,  $\lambda = \Gamma x^{\frac{3n-1}{2}} \sqrt{\frac{c^3(n+1)}{\nu}}$ ,  $R = \frac{4\sigma^* T_\infty^3}{k k^*}$  denotes the Prandtl Number, velocity ratio parameter, Williamson parameter and the radiation parameter respectively. The skin friction coefficient  $C_f$  and the local Nusselt number  $Nu_x$  at the stretching surface defined as

$$C_f = \frac{\tau_w}{\rho U_w^2} \quad Nu_x = \frac{x q_w}{k(T_\infty - T_m)} \quad (12)$$

Where  $\tau_w$  and  $q_w$  are the shear stress along the sheet and the heat flux from the surface which are given by

$$\tau_w = \mu \left( \frac{\partial u}{\partial y} + \frac{\Gamma}{\sqrt{2}} \left( \frac{\partial u}{\partial y} \right)^2 \right)_{y=0}, \quad q_w = -k \left( \frac{\partial T}{\partial y} \right)_{y=0} \quad (13)$$

$$\begin{aligned} \text{and } Re_x^{\frac{1}{2}} C_f = \left( f'' + \frac{\lambda}{2} f'^2 \right)_{y=0}, \quad Re_x^{\frac{1}{2}} Nu_x = \\ - \left( 1 + \frac{4R}{3} \right) \theta'(0) \end{aligned}$$

where  $Re_x^{\frac{1}{2}} = \frac{U_w x}{\nu}$  is local Reynolds number

### 3. RESULTS AND DISCUSSION

The transformed momentum and energy Eqs.(9)–(10) subjected to the boundary conditions Eqs. (11) were numerically solved by using Keller Box method. Figs. 2-6 are plotted for the velocity and temperature profiles for different values of governing parameters. In order to find the accuracy of our work, a comparison has been made with the previous results of Nadeem and Hussain [40], Gorla and Sidawi [41] and we obtained excellent agreement which are displayed in Table 1. Moreover, the values of skin friction coefficient and local Nusselt number for different parameters are given in Tables 2 and 3.

**Table 1. Comparison for viscous case  $-\theta'(0)$  with Pr for  $\lambda = \epsilon = R = 0$**

Pr	Present result	Nadeem and Hussain [40]	Golra and Sidawi [41]
0.07	0.06602	0.066	0.066
0.2	0.1691	0.169	0.169
0.7	0.45392	0.454	0.454
2	0.91136	0.911	0.911

The influence of velocity ratio parameter  $\varepsilon$  and Williamson parameter  $\lambda$  are displayed in Figs. 2 and 3. The behavior of  $\varepsilon$  (which denotes the ratio of free stream velocity to the velocity of the stretching sheet) on the velocity field can be observed from Fig. 2. The velocity of the fluid and the boundary layer thickness increases when free stream velocity is less than the velocity of the stretching sheet with an increase in  $\varepsilon$ . However when free stream velocity exceeds the velocity of the stretching sheet, the velocity of the fluid increases whereas the boundary layer thickness decreases with an increase in  $\varepsilon$ .

Fig 3 shows the variation of Williamson parameter  $\lambda$  on velocity profile. It can be observed that velocity decreases with increase in Williamson fluid parameter  $\lambda$ ; because after increasing Williamson fluid parameter  $\lambda$  the fluid offers more resistance to flow which decreases velocity.

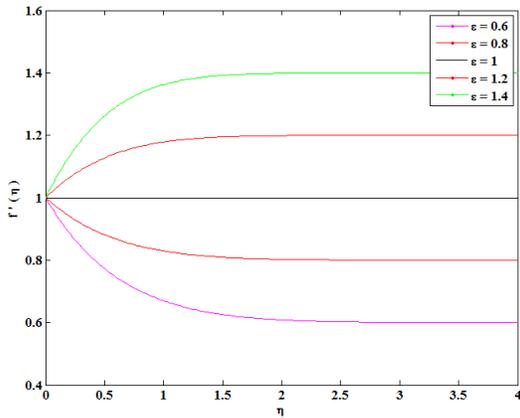


Fig. 2. Velocity profile for varying epsilon

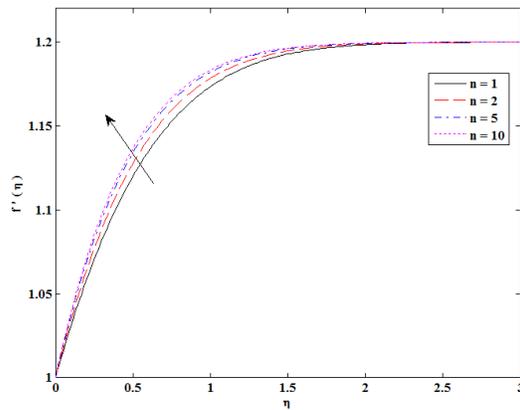


Fig. 4. Velocity profile for varying nonlinear stretching parameter

Fig. 4 shows the effect of nonlinear stretching parameter on velocity profile. The velocity of the fluid increases with the increase in nonlinear stretching parameter. As a result the momentum boundary layer thickness increases. From the graph no specific variations were observed when  $n$  is large. This is because of the term  $\frac{2n}{n+1}$  which approximately approaches to 2 when  $n$  reaches infinity. Therefore the observations for the large values of  $n$  is not a study of interest.

Fig. 5 shows the temperature profile for various values of Pr. It is clear that the dimensionless parameter  $\theta$  decreases with the increase in Prandtl number. Since the Prandtl number is the ratio of momentum diffusivity to thermal diffusivity; it reduces the thermal boundary layer thickness. In general the Prandtl number is used in heat transfer problems to reduce the relative thickening of the momentum and the thermal boundary layers.

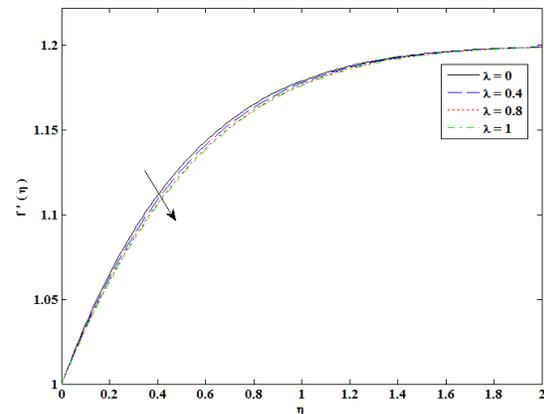


Fig. 3. Velocity profile for varying lambda

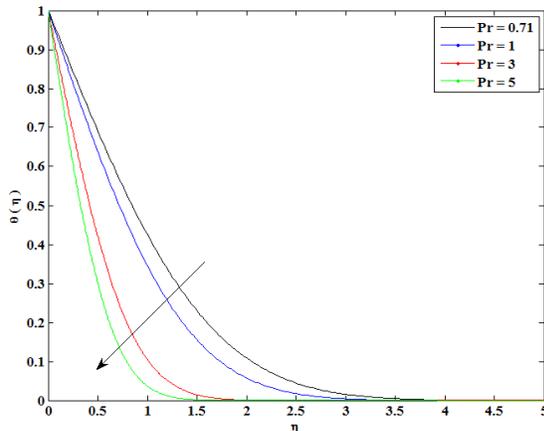


Fig. 5. Temperature profile for various values of Pr

Fig. 6 represent the temperature profiles for various values of the thermal radiation parameter R. It can be observed that an increase in the thermal radiation parameter R produces a significant increase in the thickness of the thermal boundary and so the temperature distribution increases. The effect of R is to enhance the heat.

Table 3 shows the variations of skin friction coefficient  $Re_x^{\frac{1}{2}}C_f$  for various values of  $\lambda$ ,  $\epsilon$  and n. It can be observed from the table that the values of skin friction coefficient increases when Williamson parameter, nonlinear stretching parameter n and velocity ratio parameter  $\epsilon$  increases. Table 3 presents the variation in Local Nusselt number with respect to various flow parameters. It shows that heat transfer rate  $-\theta'(0)$  decreases for radiation parameter R but for the Prandtl number  $-\theta'(0)$  increases.

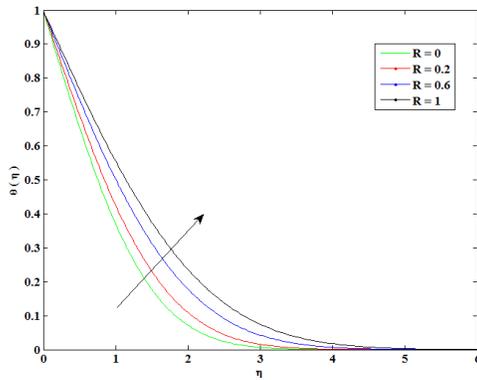


Fig. 6. Temperature profile for various values of R

Table 2. Computed values of skin friction coefficient  $Re_x^{\frac{1}{2}}C_f$  for various values of  $\lambda$ ,  $\epsilon$  and n

$\lambda$	$\epsilon$	n	$Re_x^{\frac{1}{2}}C_f$
0			0.37936
0.1			0.4248
0.2			0.4705
0.3			0.51643
	1.5		1.06418
	1		0.1
	0.8		0.24303
	0		1.09675
		1	0.4307
		5	0.5063
		10	0.52153

Table 3. Computed values of Local Nusselt number  $-\theta'(0)$  for various values of Pr and R

Pr	R	$-\theta'(0)$
1		0.74721
2		1.04793
3		1.27723
	0.1	0.66742
	0.5	0.55277
	1	0.46885

#### 4. CONCLUSION

In the present study we have investigated the stagnation point flow of a Williamson fluid over a nonlinearly stretching sheet with thermal radiation by employing a finite difference technique known as Keller Box method. The important findings are concluded as follows.

- With an increase in Williamson fluid parameter  $\lambda$ , the velocity of the fluid decreased whereas the Skin friction coefficient increased.
- The velocity of the fluid and the boundary layer thickness increases for  $\epsilon < 1$  and velocity increases and the boundary layer thickness decreases for  $\epsilon > 1$  with an increase in  $\epsilon$ .
- Nonlinear stretching parameter increases the flow field velocity.
- The thermal boundary layer thickness decreases with the effect of Prandtl number but the opposite effect is observed with the radiation parameter.
- The wall temperature gradient increases with an increase in Pr but decreases with radiation parameter R.

#### COMPETING INTERESTS

Authors have declared that no competing interests exist.

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