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Maxillary and Mandibular Arch Perimeter Prediction Using Ramanujan's Equation for the Ellipse-*In vitro* Study

Rahul Aghera[1*], Hina Desai[1], Padmaja Sharma[1], Nirav Dholakiya[1] and Nishil Agrawal[1]

[1]Department of Orthodontics and Dentofacial Orthopaedics, Manubhai Patel Dental College, Hospital and Oral Research Institute, Vadodara, India.

Authors' contributions

This work was carried out in collaboration between all authors. Author RA wrote the study protocol, carried out experimental part of the study and wrote the first draft of the study. Authors HD and PS designed the study and carried out the manuscript corrections. Authors ND and NA searched for the literature and carried out the analysis. All authors read and approved the final manuscript.

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Original Research Article

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ABSTRACT

Introduction: Arch expansion in the molar region or incisor proclination is an effective method to resolve arch length discrepancy. Predicting the change in arch perimeter by arch expansion in the molar region or incisor proclination is important in deciding during treatment planning if the removal of teeth will be required.
Purpose: The primary objective of this study was to determine if there is a mathematical correlation between Ramanujan's equation for the perimeter of an ellipse and the maxillary and mandibular arch perimeter. The secondary goal was to predict the gain in arch perimeter by arch expansion in molar region and incisor proclination using same equation for non-extraction cases.
Materials and Methods: 55 maxillary and 55 mandibular well-aligned diagnostic casts of untreated patients with class I molar relationships were used. Arch perimeter was measured

**Corresponding author: E-mail: dr.rahulaghera@gmail.com;*

using light wire and was compared with the calculated arch perimeter obtained by using Ramanujan's equation.
Results: A strong correlation was found between measured and calculated arch perimeters with 1.5% error in maxillary arch and 1.7% error in mandibular arch. The average gain in arch perimeter by 1 mm molar expansion was 0.73 mm in maxilla and 0.74 mm in the mandibular arch. Average gain in arch perimeter by 1mm incisor proclination was 1.67 mm in maxilla and 1.65 mm in the mandible.
Conclusion: Ramanujan's equation for an ellipse can be used to calculate the arch perimeter in the maxillary arch with a 1.5% chance of error and a 1.7% chance of error in mandibular arch. The equation can also be used to predict change in arch perimeter by molar expansion and incisor proclination.

Keywords: Arch expansion; arch perimeter; Ramanujan's equation.

1. INTRODUCTION

There are several factors, including space requirements, that should be considered when deciding whether to treat a malocclusion with or without extractions. In a borderline case, expansion can be used to solve the space deficiency problem and treat the malocclusion without extraction, if the patient's condition permits the expansion. The prediction of change in arch perimeter for a given amount of expansion is beneficial in planning the treatment of patients who need expansion and can facilitate non-extraction orthodontic treatment [1].

Usually space is required in a dental arch for correction of features of a malocclusion such as crowding, overjet reduction, levelling of the curve of Spee or correction of incisor inclination and angulation [2]. There are various methods in orthodontics to treat the Arch Length Deficiency (ALD), such as expanding the arch, proclining the anterior incisors, distalizing the posterior dentition, reducing teeth interproximally, or extracting teeth. The decision of the treatment modality depends on the occlusion, facial esthetics, and stability of the result. Expanding the arch and proclining the anterior incisors can be an effective way of gaining arch perimeter when retaining teeth is indicated [3].

An accurate prediction of change in arch perimeter when proclining anteriors or expanding the arch in a borderline extraction patient is necessary [3]. Studies have been carried out to determine the relationship between several arch measurements that are important in orthodontic treatment. As reported by some authors, predicting the change in arch length (AL) as a consequence of transverse expansion may be helpful in orthodontic treatment planning [4-7]. Some investigators have attempted to predict the peripheral arch length changes related to arch width expansion. Adkins et al. [5] concluded that there was a 4.7 mm average increase in arch perimeter for a 6.5 mm average molar expansion. These results were obtained by recording dental landmarks of 21 orthodontic patients who had undergone rapid palatal expansion and did not attempt to account for any related canine expansion. Rickets et al. [6] established a relationship between change in arch perimeter from canine expansion and molar expansion. He reported that each 1 mm of canine expansion leads to 1 mm increase in arch perimeter, and 1 mm of molar expansion leads to 0.25 mm increase in arch perimeter.

Germane et al. [7] quantified the change in arch perimeter related to orthodontic expansion based on the premise that the mathematic spline function describes the dental arch with acceptable accuracy. When the canine width and incisor positions were held constant, an initial 1 mm increase in molar expansion resulted in an approximately 0.27 mm increase in perimeter, an additional millimeter increase resulted in an additional 0.31 mm, and a 5 millimeter increase of molar expansion resulted in 0.41 mm increase in arch perimeter. When the incisor positions were fixed, each 1 mm of canine expansion resulted in 0.73 mm increase in arch perimeter. Motoyoshi conducted a finite element method (FEM) study to estimate the increase in arch perimeter associated with mandibular lateral expansion. He concluded that 1mm of increase of arch width resulted in 0.37 mm of increase in arch perimeter [8]. Currier conducted a study in relation to human dental arch form. He concluded that the human arch form resembled the geometrically in form of an ellipse [9].

The calculation of the perimeter of an ellipse can be calculated using Ramanujan's equation originally formulated in 1914 [10]. Chung et al. conducted a study in which they compared the

measured value of a maxillary arch perimeter with calculated value of maxillary arch perimeter using Ramanujan's equation. They concluded that there was high level of correlation between measured value and calculated value with an error of 1.2%.They also used the same formula to predict the change in maxillary arch perimeter by expansion or proclination of anteriors. They found that there was 0.73 mm of gain in arch perimeter for every 1mm of molar expansion and 1.66 mm for every 1 mm of proclination of anteriors [3].

In Chung's study only the maxillary arch was used for computing the accuracy of using Ramanujan's equation for calculating the arch perimeter. Hence this study was carried out to determine if there is a quantitatively mathematical correlation between Ramanujan's equation for the perimeter of an ellipse and the perimeters of the maxillary and mandibular archs. The secondary objective was to apply the equation in predicting the arch perimeter gained by expansion of the molars or proclination of the incisors.

2. MATERIALS AND METHODS

55 sets of diagnostic casts (maxillary and mandibular) of untreated patients with well-aligned dentition who had received no prior orthodontic treatment were selected from Department of Orthodontics and Dentofacial Orthopedics, Manubhai Patel Dental College Vadodara

2.1 Inclusion Criteria

1. Well-aligned arch;
2. A full dentition from first molar to first molar;
3. Mild spacing is accepted.

2.2 Exclusion Criteria

1. Severely crowded arch;
2. Patient who has undergone prior orthodontic treatment;
3. Deciduous or mixed dentition.

Linear measurements were obtained from the midbuccal surfaces of the distobuccal cusps of the first molars with a digital caliper (TOLEXO-electronic digital Vernier caliper with LCD display screen, Mumbai, India) for maxillary and mandibular arch. The perpendicular distance was then measured from the line intersecting the distobuccal cusps of the first molars and the facial surfaces of the central incisors for maxillary and mandibular arch respectively using a straightedge ruler and the caliper. Measurement of the arch perimeter was made directly on each model from the distobuccal cusps of the first molars with a light-gauge wire that contacted the midbuccal surface of each tooth for maxillary and mandibular arch. The light-gauge wire was marked at the distobuccal cusps of the first molars and then the wire was straightened. The marks made on the light-gauge wire were measured with the caliper.

The data was then inserted into Ramanujan's equation for the perimeter of an ellipse.

$$\pi(a+b)\{1+(3h/(10+\sqrt{(4-3h)})\}$$

Where $h= (a-b)^2/ (a+b)^2$

The value for "a" was taken to be the perpendicular distance measured from the line intersecting the distobuccal cusps of the first molars and the facial surfaces of the central incisors. The value for "b" was taken to be the linear measurement taken from the midbuccal surfaces of the distobuccal cusps of the first molars divided in half.

The value was calculated with Ramanujan's equation for the perimeter of an ellipse which was then compared with the value measured directly on each model from the distobuccal cusps of the first molars using a light-gauge wire that contacted the midbuccal surface of each tooth.

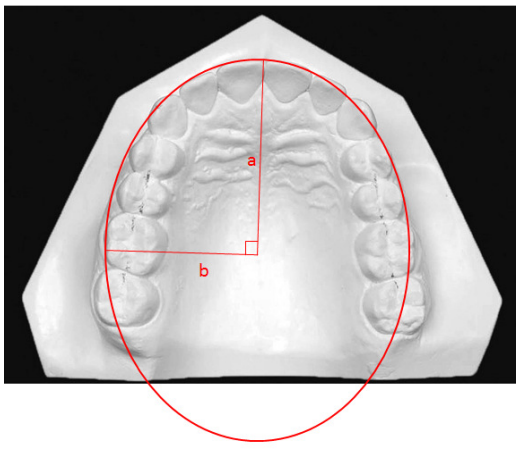

Fig. 1. Diagram of an ellipse, where *a* is the semimajor axis, and *b* is the semiminor axis. An ellipse was fit on the maxillary dental arch schematically

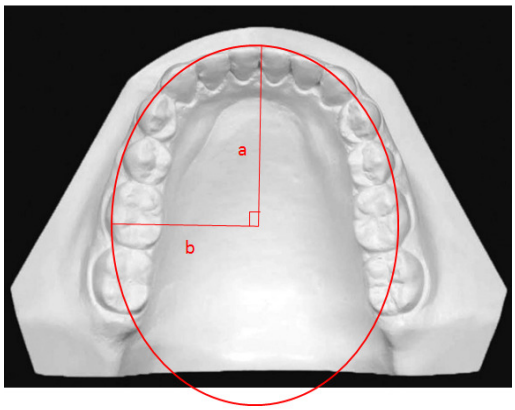

Fig. 2. Diagram of an ellipse, where *a* is the semimajor axis, and *b* is the semiminor axis. An ellipse was fit on the mandibular dental arch schematically.

2.3 Statistical Analysis

Pearson correlation was used to determine the correlation between the measured value of arch perimeter and calculated value of arch perimeter. Mean error in percentage between measured value and calculated value was also recorded. Mean and standard deviation was calculated to obtain the average change in arch perimeter after the molar expansion and incisor proclination.

3. RESULTS

3.1 Maxillary Arch

The results (Fig. 3) showed strong correlation between measured perimeter and calculated perimeter of maxillary arch (Pearson correlation = 0.93, $p<0.01$). 1.5% error was found between measured perimeter and calculated perimeter of maxillary arch using Ramanujan's equation. Also using the same equation to calculate the change in arch perimeter after expansion and incisor proclination, results showed that for every 1 mm of molar expansion there was 0.73 mm ± 0.01 mm change in arch perimeter and for every 1 mm of incisor proclination there was 1.67 mm ± 0.02 mm change in arch perimeter.

3.2 Mandibular Arch

The results (Fig. 4) showed strong correlation between measured perimeter and calculated perimeter of mandibular arch (Pearson correlation = 0.93, $p<0.01$). 1.7% error was found between measured perimeter and calculated perimeter of maxillary arch using Ramanujan's equation. Also using the same equation to calculate the change in arch perimeter after expansion and incisor proclination, results showed that for every 1 mm of molar expansion there was 0.74 mm ± 0.02 mm change in arch perimeter and for every 1 mm of incisor proclination there was 1.65 mm ± 0.03 mm change in arch perimeter.

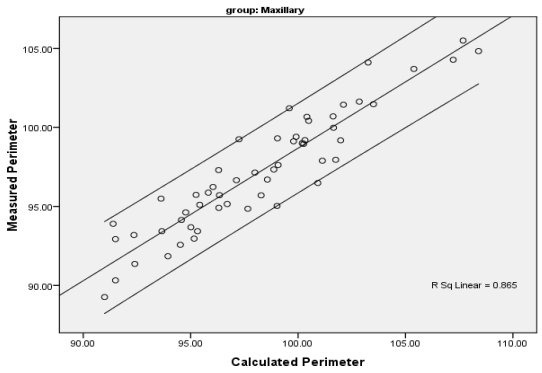

Fig. 3. The measured perimeter of the maxillary arch was plotted against the calculated perimeter. Pearson correlation = 0.93. Correlation is significant to the 0.01 level

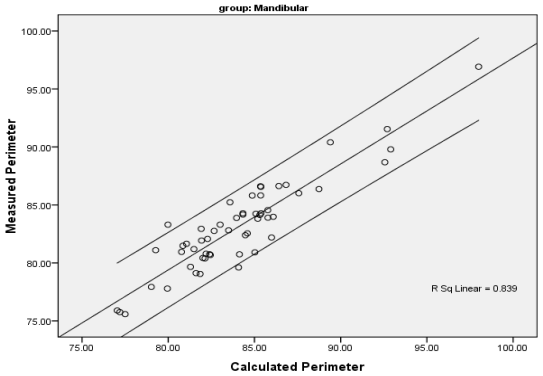

Fig. 4. The measured perimeter of the mandibular arch was plotted against the calculated perimeter. Pearson correlation = 0.91. Correlation is significant to the 0.01 level

4. DISCUSSION

In many orthodontic cases, expansion or proclination of anterior teeth is an effective method to resolve arch length discrepancy in non-extraction treatment planning. In this study, we have used Ramanujan's equation of an ellipse to calculate the arch perimeter of the

maxillary as well as the mandibular arch. Using the same equation alteration in arch perimeter after molar expansion and incisor proclination also was calculated. As demonstrated by Chung et al. [4], Ramanujan's equation for the the perimeter of an ellipse can be used to calculate the maxillary arch perimeter with acceptable accuracy of 1.2% error. In this study, we found that there was an average error of 1.5% for maxillary arch and a 1.7% acceptable error in mandibular arch. Therefore, Ramanujan's equation appears to be an effective method for calculating the arch perimeter of both the maxillary arch as well as the mandibular arch.

Table 1. Measurements in maxillary arch

Sr. no.	U6 to u6 (mm)	U1 to u6 (mm)	Calculated perimeter (mm)	Measured perimeter (mm)	Error (%)	1 mm molar expansion (mm)	1 mm incisor proclination (mm)
1	56.25	33.11	96.34	95.71	0.65	0.75	1.63
2	58.31	39.04	107.68	105.5	2.06	0.72	1.68
3	53.58	34.29	96.3	97.3	1.02	0.74	1.67
4	47.01	34.25	91.5	90.32	1.3	0.7	1.7
5	50.11	36.58	97.66	94.85	2.96	0.7	1.7
6	53.9	35.51	98.57	96.7	1.93	0.73	1.67
7	54.19	37.42	101.98	99.18	2.82	0.72	1.69
8	55.2	34.59	97.99	97.14	0.87	0.74	1.66
9	60.6	35.09	102.85	101.63	1.2	0.75	1.63
10	51.48	34.29	94.77	94.62	0.15	0.73	1.68
11	52.46	34.15	95.25	95.73	0.5	0.73	1.67
12	48.56	35.04	93.94	91.85	2.27	0.71	1.71
13	51.21	32.38	91.39	93.9	2.67	0.73	1.65
14	50.07	34.21	93.62	95.49	1.95	0.71	1.68
15	53	36.86	100.19	99	1.2	0.71	1.69
16	52.14	32.04	91.5	92.93	1.53	0.76	1.67
17	55	35.33	99.07	97.62	1.48	0.74	1.67
18	53.85	34.67	97.13	96.66	0.48	0.73	1.66
19	51.6	34.38	95.01	93.68	1.41	0.72	1.68
20	52.25	37.01	99.91	99.41	0.5	0.7	1.69
21	54.22	36.52	100.49	100.42	0.06	0.73	1.69
22	54.08	36.16	99.79	99.12	0.67	0.72	1.68
23	53.44	33.31	94.57	94.14	0.45	0.75	1.66
24	57.64	31.83	95.32	93.43	2.02	0.77	1.62
25	53.93	36.6	100.41	100.67	0.25	0.72	1.69
26	50.84	35.32	96.04	96.23	0.19	0.72	1.69
27	54.06	37.34	101.75	97.95	3.87	0.73	1.7
28	52.09	37.68	100.92	96.48	4.6	0.71	1.71
29	56.59	37.15	103.26	104.11	0.81	0.73	1.67
30	55.73	39.47	107.22	104.28	2.16	0.72	1.7
31	55.25	33.25	95.82	95.87	0.05	0.74	1.64
32	53.63	35.81	98.87	97.34	1.57	0.72	1.68
33	49.8	38.26	100.32	99.19	1.13	0.78	1.72
34	52.81	32.25	92.36	93.19	0.89	0.74	1.64
35	53.48	38.62	103.51	101.47	2.01	0.71	1.71
36	57.16	32.21	95.42	95.1	0.49	0.76	1.62
37	53.18	34.47	96.31	94.91	1.47	0.73	1.67
38	49.21	38.08	99.59	101.21	1.62	0.7	1.73
39	55.11	36.82	101.63	100.7	0.92	0.73	1.68
40	55.91	38.71	105.39	103.7	1.62	0.72	1.69
41	56.28	34.28	98.28	95.7	2.69	0.75	1.65
42	59.87	34.98	102.11	101.44	0.66	0.76	1.63
43	51.02	33.07	92.4	91.36	1.13	0.73	1.67
44	57.45	39.87	108.4	104.83	3.4	0.77	1.74

Sr. no.	U6 to u6 (mm)	U1 to u6 (mm)	Calculated perimeter (mm)	Measured perimeter (mm)	Error (%)	1 mm molar expansion (mm)	1 mm incisor proclination (mm)
45	53.26	33.36	94.52	92.57	2.1	0.74	1.66
46	50.41	32.5	90.99	89.26	1.93	0.74	1.67
47	54.21	34.59	97.25	99.25	2.01	0.74	1.67
48	52.11	36.56	99.04	99.31	0.27	0.71	1.69
49	55.7	36.26	101.13	97.89	3.49	0.74	1.67
50	55.4	32.78	95.16	92.96	2.36	0.76	1.64
51	54.18	35.66	99.02	95.03	4.19	0.73	1.68
52	52.19	33.32	93.66	93.43	0.24	0.74	1.67
53	54.92	36.08	100.26	98.96	1.31	0.73	1.68
54	50.87	35.7	96.7	95.15	1.62	0.71	1.69
55	54.9	36.92	101.65	99.98	1.67	0.73	1.69
					1.543	0.731	1.675

Table 2. Measurements in mandibular arch

Sr. no.	L6 to l6 (mm)	L1 to l6 (mm)	Calculated perimeter (mm)	Measured perimeter (mm)	Error (%)	1 mm molar expansion (mm)	1 mm incisor proclination (mm)
1	50.67	33.55	92.9	89.79	3.46	0.76	1.71
2	48.91	27.12	81.06	81.65	0.72	0.76	1.61
3	51.92	36.02	98	96.92	1.11	0.66	1.69
4	46.8	28.75	82.13	80.39	2.16	0.74	1.65
5	47.68	28.44	82.28	82.07	0.25	0.75	1.64
6	40.96	28.53	77.5	75.58	2.54	0.72	1.69
7	45.23	32.68	87.57	86.01	1.81	0.71	1.7
8	46.25	30.2	84.13	80.74	4.19	0.72	1.66
9	51.39	26.79	82.44	80.68	2.18	0.78	1.59
10	49.83	29.31	85.31	84.14	1.39	0.75	1.63
11	54.08	31.88	92.7	91.54	1.26	0.76	1.64
12	47.16	30.98	86.09	83.98	2.51	0.73	1.68
13	47.44	27.14	79.98	83.3	3.98	0.76	1.62
14	41.34	30.72	81.49	81.19	0.36	0.71	1.72
15	45.09	31.15	84.87	85.81	1.09	0.72	1.7
16	42.57	28.9	79.27	81.1	2.25	0.73	1.69
17	47.55	30.37	85.36	85.82	0.53	0.73	1.66
18	47.31	28	81.29	79.66	2.04	0.73	1.62
19	47.84	27.44	80.77	80.95	0.22	0.76	1.63
20	48.43	30.36	85.99	82.19	4.62	0.74	1.66
21	45	29.48	82.01	80.43	1.96	0.73	1.68
22	48.85	31.83	88.74	86.37	2.74	0.73	1.67
23	47.87	29.61	84.33	84.28	0.05	0.75	1.66
24	46.36	31.14	85.78	83.9	2.24	0.73	1.68
25	52.97	27.87	85.38	84.28	1.31	0.78	1.6
26	46.64	28.7	81.93	81.93	0	0.74	1.65
27	43.89	30.07	82.19	80.79	1.73	0.72	1.69
28	47.73	31.17	86.83	86.73	0.1	0.72	1.66
29	46.34	30.44	84.59	82.55	2.47	0.73	1.68
30	49.96	27.83	83.01	83.3	0.34	0.76	1.62
31	50.36	31.56	89.4	90.4	1.09	0.74	1.66
32	45.9	29.02	81.91	82.94	1.24	0.74	1.66
33	42.24	30.88	82.4	80.75	2.04	0.7	1.71
34	48.74	26.51	79.95	77.79	2.77	0.77	1.61
35	47.57	30.26	85.19	83.81	1.64	0.73	1.66

Sr. no.	L6 to l6 (mm)	L1 to l6 (mm)	Calculated perimeter (mm)	Measured perimeter (mm)	Error (%)	1 mm molar expansion (mm)	1 mm incisor proclination (mm)
36	49.95	28.74	84.47	82.39	2.52	0.76	1.63
37	47.58	28.08	81.61	79.12	3.14	0.76	1.64
38	43.87	31.94	85.35	86.61	1.45	0.7	1.7
39	49.71	28.77	84.33	84.17	0.19	0.76	1.64
40	49.73	30.03	86.41	86.63	0.25	0.75	1.64
41	50.46	28.84	85.02	80.91	5.07	0.76	1.63
42	58.28	29.79	92.56	88.68	4.37	0.79	1.59
43	45.31	27.54	79.02	77.93	1.39	0.75	1.65
44	49.69	29.66	85.77	84.55	1.44	0.75	1.64
45	45.82	30.29	83.96	83.88	0.09	0.73	1.68
46	45.79	26.2	77.19	75.76	1.88	0.76	1.63
47	47.38	30.45	85.37	86.56	1.37	0.74	1.67
48	46.6	28.67	81.85	79.05	3.54	0.74	1.65
49	50.54	27.86	83.5	82.81	0.83	0.77	1.62
50	47.84	28.6	82.66	82.76	0.12	0.75	1.64
51	49.94	28.5	84.08	79.61	5.61	0.76	1.62
52	47.02	29.53	83.58	85.23	1.93	0.74	1.66
53	43.18	27.29	77.04	75.9	1.5	0.74	1.66
54	48.92	26.98	80.85	81.48	0.77	0.76	1.61
55	49.51	29.32	85.08	84.24	0.99	0.75	1.64
					1.797091	0.742	1.653091

In this study the gain in arch perimeter in maxillary and mandibular arch by expansion of molars and proclination of anteriors was also calculated using the Ramanujan's equation. Results showed that in maxillary arch every 1 mm of molar expansion resulted in increase of 0.73 mm in arch perimeter and every 1 mm of incisor proclination resulted in increase of 1.67 mm in arch perimeter. In mandibular arch every 1 mm of molar expansion resulted in increase of 0.74 mm in arch perimeter and every 1 mm of incisor proclination resulted in increase of 1.65 mm in arch perimeter. These results correlate well with the study of Chung et al. [4] where they found that every 1 mm of molar expansion resulted in 0.73 mm gain in arch perimeter and every 1 mm incisor protrusion resulted in 1.66 mm gain in maxillary arch perimeter.

Adkin's et al. [5] found that there was an average increase of 4.7 mm in arch perimeter for an average molar expansion of 6.5mm which comes to 0.72 mm of an increase in arch perimeter for every 1mm of molar expansion which is similar to our study. Hnat et al. [11] reported that when molar expansion was done by 6 mm, there was 5.4 mm alteration in arch perimeter, giving an average of 0.9 mm gain in arch perimeter for every 1 mm molar expansion which is comparatively higher than the result obtained in our study.

Few studies have also been reported to quantify the change in arch perimeter in mandibular arch after mandibular expansion. Motoyoshi et al. [8] conducted a study to estimate the increase in arch perimeter associated with mandibular lateral expansion using 3D finite element method. He concluded that there was 0.37 mm gain in arch perimeter for every 1 mm increase in arch width which is less as compared to our study. Similarly Sabrina et al. [12] conducted a study using the mathematical geometric model to calculate alteration in arch length by proclination of anteriors and concluded that for every 1 mm of incisor proclination there is gain of 1.21 mm to 2.07 mm arch perimeter. In our study results showed that in the mandibular arch, every 1 mm of incisor proclination resulted in 1.65 mm of gain in arch perimeter which is within the range of results reported in the previous study. Steiner [13] concluded that every 1 mm of incisor proclination resulted in 2.07 mm of gain in arch perimeter which is higher as compared to results obtain in our study.

Germane et al. [7] concluded that change in arch perimeter by incisor proclination was four times as compared to change in arch perimeter by molar expansion. In our study, results showed that it was just over twice the change in arch perimeter by incisor proclination compared to molar expansion.

O'Higgins et al. [2] and Akkaya et al. [14] found that there was 0.65 mm increase for every 1 mm molar expansion in rapid maxillary expansion and 0.60 mm for slow maxillary expansion. These results are similar to results obtained in our study.

The reason for difference in results between our study and other studies could be because of difference in methodologies. Another probable reason could be the population, as this study was carried out in an Indian population.

Future studies might include selecting the samples based on various demographic areas. Also in the present study only Class I molar relation was considered during selection of samples. Therefore further studies can be carried out on various different malocclusions to evaluate the accuracy of calculating arch perimeter using Ramanujan's equation for ellipse.

5. CONCLUSION

Ramanujan's equation for an ellipse can be used for calculating the arch perimeter of maxillary and mandibular arch with a 1.5% error in maxillary arch and 1.7% error in mandibular arch. In maxillary arch, the predicted amount of gain in arch perimeter was 0.73 mm for every 1 mm molar expansion and 1.67 mm for every 1 mm incisor proclianation. In mandibular arch, the predicted amount of gain in arch perimeter was 0.74 mm for every 1 mm molar expansion and 1.65 mm for every 1 mm incisor proclination.

CONSENT

It is not applicable.

ETHICAL APPROVAL

Institutional ethics committee (iec) for research Manubhai Patel dental college & hospital & ori, Vadodara. Ref. No.:- iec/mpdc_67/ ortho-14/15.

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COMPETING INTERESTS

Authors have declared that no competing interests exist.

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