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Fine Structure Constant is Related to Effective Nuclear Charge and Bohr's Radius for Any Atom

Ikechukwu Iloh Udema[1*]

[1]Owa Alizomor Mixed Sec. Sch. Owa Alizomor, Ika North East/Ude International Concepts LTD (862217), B. B. Agbor, Delta State, Nigeria.

Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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ABSTRACT

Objectives: The objectives of this research were to show that fine structure constant (α) is an intrinsic property of any matter (element) and 2) to relate it to other physical property of elements such as Bohr's radius of any element and effective nuclear charge.
Methods: The method is essentially theoretical which entailed the substitution of experimental data, ionization energy into derived equations in order to verify the universality of α as an intrinsic property of elemental matter.
Results: The results for hydrogen showed that the effective nuclear charge, Z_{eff} ($\cong$ 1) and Bohr's radius obtained by calculation after substituting appropriate average ionization energy into different equations were very similar. The results for other elements that are non-hydrogenic showed similar similarity of values regardless of the equation used. $Z_{\text{eff}} = \frac{n}{\alpha c}\sqrt{\frac{2E_{\text{I}}}{m_e}}$: Where the principal quantum number, $n \geq 1$, E_{I} is the average ionization energy of any element, and m_e is the mass of an electron.
Conclusion: The fine structure constant is an intrinsic property of matter. There is a relationship between the fine structure constant and well known periodic properties of elemental matter, effective nuclear charge, average ionization energy, and Bohr's radius for any atom, other than hydrogenic atoms.

**Corresponding author: E-mail: udema_ikechukwu99@yahoo.com;*

Keywords: Fine structure constant; periodic properties of elements; hydrogen atom; non-hydrogenic atom; Bohr's radii.

1. INTRODUCTION

The term fine structure constant (α) has been subjected to a lot of misgivings and interpretation, which are subjective, mystical and strangely scientific in nature. It does not seem to have any useful value or significance. Treating α as mere figure is subjective and amount to trivialization in nature. If complete science or idea is one that is not just testable but subject to mathematical formalism, then the strong mathematical formulations which give an impression that α is a difficult concept is unnecessarily strange. As a mystery it implies that it is not subject to human-level activity called scientific process. The misgivings, misinterpretations and mysticism associated with α can be attested to in part as follows. "…It has been a mystery ever since it was discovered more than fifty years ago, and all good theoretical physicists put this number up on their wall and worry about it. Immediately you would like to know where this number for a coupling comes from: is it related to π or perhaps to the base of natural logarithms? Nobody knows. It's one of the greatest damn mysteries of physics: a magic number that comes to us with no understanding by man. You might say the 'hand of God' wrote that number, and 'we don't know how He pushed his pencil.' We know what kind of a dance to do experimentally to measure this number very accurately, but we don't know what kind of dance to do on the computer to make this number come out, without putting it in secretly!" [1]. *Perhaps Indian or Italian dance style only may be the answer; but this must not be without time consciousness*!

However, α is said to come from spectroscopy where it is used in the development of J.J. Ballmer's series of a spectral lines into multiple very close components [2]. Other uses of α are: various determinations such as interferometry and Bloch oscillations, the neutron Compton wavelength measurement, unfamiliar areas such as AC Josephson effect and quantum Hall effect in condense matter physics *etc* [3]. Also, it has application in the determination of the strength of the electromagnetic interaction and it was according to Sherbon [3] initially introduced by Sommerfeld [4]. Fine structure constant, α is defined as a ratio of the constant, *e*, *h*, and *c*, having the expression: $\alpha = e^2/\hbar c$ where the parameters are respectively, charge of an electron, Planck's constant and speed of light in free space and $\hbar = h/2\pi$. [2,5,6]. But that is an equation yielding upon calculation 4.061407 exp (-13)/C^2/Jm which is very different from the true value.

As observed by Sherbon [3], Sommerfeld [4] defined α as the ratio of the speed of the electron in the ground state of the Bohr's hydrogen atom to the speed of light. This is highly instructive or informative. The notion of negative kinetic energy has been questioned [7]. It is described as being nonsensical [7]. This is understandable considering the fact that the farther away the electron from the nucleus, the higher the potential energy but the kinetic energy decreases. The minimum energy needed to remove an electron in the outermost energy level of an atom in its ground state is the kinetic energy otherwise called average ionization energy: But $m_e c^2$ is not kinetic energy (m_e is the mass of an electron). Given that it may take some time, *t* to escape nuclear attraction completely, "the power" to remove the electron is:

$$P = -\frac{dE_I}{dt} \tag{1}$$

Thus in line with Sommerfeld definition, the expression for α may be:

$$\alpha = \sqrt[2]{\frac{2E_H}{m_e c^2}} \; (\sim 7.297666718 \text{ exp } (-3)) \tag{2}$$

Definition as advanced by Chakeres [8] is a little bit confusing. The average ionization energy, E_H, of hydrogen is conceptually related to the change in the velocity of an electron from the annihilation velocity of *c* to velocity (*v*), αc. Again, this means that c/v ($\equiv c/\alpha c$) = $1/\alpha$) where $v = (2E_I/m_e)^{½}$. Thus the correct expression is:

$$\frac{c}{v\sqrt{2}} = 137.0303048 \tag{3}$$

where c = 2.9979 exp (+8) m/s; m_e = 9.103 exp (-31) kg; E_I for hydrogen is 1312000 J/mol.

It is against the backdrop of different opinions held regarding what α stands for and for the fact that it seems to be tied only to hydrogen, the objectives of this research are 1) to show that α

is an intrinsic property of any matter (element) and 2) to relate it to other physical property of elements such as Bohr's radius of any element and effective nuclear charge.

2. FURTHER THEORETICAL DEVELOPMENT

2.1 Review of Literature on Mathematical Models of Fine Structure Constant

The purpose of this section is the derivation or formulation of verifiable mathematical models that can relate α to other physical properties, namely ionization energy of not just hydrogenic atoms but multi-electron atoms and radius of any atom. Meanwhile, according to Dattoli [1] arguments related to the Dirac monopoles may allow the derivation of independent formulae for the definition of fine structure constant, α. Anyone who has deep understanding of higher mathematics may consult the paper by Preskill [9] who envisaged a magnetic monopole as a semi-infinitely long, infinitesimally thin solenoid. How argument related to Dirac monopole can allow the derivation of formula for α is not certain. But an example of what seems to be the formula is what according to Dattoli [1] has been described as Wyler [10] "marvelous" formula:

$$\alpha_{\mathrm{QED}} = \boldsymbol{\alpha} = \frac{9}{16\pi^3}\sqrt[4]{\frac{\pi}{5!}} = 7.289276981\ \exp(-3) \quad (4)$$

where QED is the not-too-familiar quantum electrodynamics.

The value indicated in Eq. (4) is slightly different from what is presented in Eq. (2). A very confusing formula which cannot be made part of the numbered equations is that described as a simple formula, which yields a remarkable numerical agreement with the experimental value, namely: $\alpha(\beta) = \frac{\pi}{\beta}\cos(\beta)\,Tanc\left(\frac{\beta}{29}\right)$ where $Tanc\,(x) = \frac{\tan(x)}{x}$ and $\beta = \frac{\pi}{137}$. Further analysis shows that $x = \frac{\beta}{29} = \frac{\pi}{137\times29}$. Therefore, $\frac{\tan(x)}{x} = 1.000000209$; $\frac{\pi\cos(\beta)}{\beta} = 136.999989$. $\therefore \alpha(\beta) = 137.0000176$. Yet, $\alpha(\beta)$ or $\alpha\left(\frac{\pi}{137}\right)$ is reported to be $\frac{1}{137.035999786699}$ [1]. Highly respectable the journal in which this result was found, it is nonetheless necessary to justify the solution indicated in literature in the light of this analysis against the backdrop of well known standard abbreviations such as tan and cos so as to give even greater respectability to the journal.

There are equations in literature that deserve being numbered in this research. Example presented in different ways by Chakeres [8] and Danescu [2] are respectively,

$$2\boldsymbol{\alpha} = \frac{c\mu_0 e^2}{h} \quad (5)$$

$$\boldsymbol{\alpha}_9^1 = \frac{\mu_0 c^2}{4\pi}.\frac{\boldsymbol{e^2}}{\hbar c} = \boldsymbol{\alpha} \quad (6)$$

However, review paper by Dattoli [1] presented theories and methods for the determination of α (sometimes written as α_{QED}): The mathematical formalism may not be too familiar though such formalism represents an expression of superior knowledge in higher mathematics, but there are some results shown in the paper that cannot be obtained from some of the equations. One of such claim in literature is: $\frac{1}{\boldsymbol{\alpha}_{\mathrm{QED}}} = \frac{1}{2\sqrt{\boldsymbol{\alpha}_{\mathrm{G}}}}\exp\left(-2\sqrt{\frac{m_{\mathrm{p}}}{m_{\mathrm{e}}}}\right) = 136.976(8)$ where $\boldsymbol{\alpha}_{\mathrm{G}} = \frac{Gm_e\, m_{\mathrm{p}}}{\hbar c}$, where *G* is Newton's gravitational constant. Given that the latest gravitational constant cited elsewhere [11] is 6.67384 (80) exp (-11) $m^3/kg.s^2$; m_p/m_e = 1836.1; $m_e.m_p$ = 1.52348025 exp (-57) kg^2; α_G = 3.21722883 exp(-42); $\frac{1}{2\sqrt{\boldsymbol{\alpha}_{\mathrm{G}}}} = 5.420069\exp(+20)$; $\exp\left(-2\sqrt{\frac{m_{\mathrm{p}}}{m_{\mathrm{e}}}}\right) = 2.469022\exp(-19)$. $\therefore \frac{1}{\boldsymbol{\alpha}_{\mathrm{QED}}} = 137.678.$ It is obvious that this result is slightly different from that observed in literature. In the same paper [1] is: $\boldsymbol{\alpha}_{\mathrm{QED}} = \frac{e^2}{4\pi}$. It is clear that $\boldsymbol{\alpha}_{\mathrm{QED}}$ does not have any other meaning other than how it was defined in literature. If so, the latter equation may not be valid considering what *e* stands for. Nonetheless there are other equations that give similar values of 'α'. Such are [11]:

$$\boldsymbol{\alpha}_{\mathrm{QED}} = \frac{9}{8\pi^4}\cdot\sqrt[4]{\frac{\pi^5}{2^4\,5!}} \quad (7)$$

$$\approx \frac{4\pi}{N(N-1)} \text{ where } N = 42 \quad (8)$$

2.2 Evidence for the Universality or Intrinsic Nature of Fine Structure Constant

It seems $\boldsymbol{\alpha}$ has always been linked to ionization of hydrogen only [8]. It is the objective of this section to show the evidence for the universality

of '**α**'. As shown elsewhere [12], the effective nuclear charge Z_{eff} is given as follows.

$$Z_{\text{eff}} = \sqrt[2]{\left(\frac{8E_{\text{I}}}{m_e}\right)} \cdot \frac{nh\varepsilon_0}{e^2} \tag{9}$$

where E_{I} m_{e}, n, ε_0, h, and e are the ionization energy per electron, mass of an electron, principal quantum number, permittivity in free space, Planck's constant, and electron charge. It can be stated emphatically that,

$$E_{\text{I}} = \frac{Z_{\text{eff}}^2 E_{\text{H}}}{n^2} \tag{10}$$

Making E_{H} subject of the formula (Eq. (8)) gives,

$$E_{\text{H}} = \frac{n^2 E_{\text{I}}}{Z_{\text{eff}}^2} \tag{11}$$

Equation (11) can be substituted into Eq. (2) to give:

$$\boldsymbol{\alpha} = \sqrt{\frac{2n^2 E_{\text{I}}}{Z_{\text{eff}}^2 m_e c^2}} \tag{12}$$

Taking the square of Eq. (9) and substitute into Eq. (12) to give:

$$\boldsymbol{\alpha} = \sqrt{2m_e\, e^4\, E_{\text{I}} \frac{n^2}{8E_{\text{I}} n^2 h^2 \varepsilon_0^2 m_e c^2}} \tag{13}$$

$$= \frac{e^2}{2h\varepsilon_0 c} \tag{14}$$

2.3 Relating Fine Structure Constant to Other Physical Constant

2.3.1 Relating fine structure constant to Bohr's radius of any atom

From what seem to be the first principle, Bohr's equation for the radius of any atom, the effective nuclear charge can be expressed as:

$$Z_{\text{eff}} = \frac{n^2 h^2 \varepsilon_0}{\pi e^2 m_{\text{e}} a} \tag{15}$$

where $a \geq a_0$ where a_0 is the usual Bohr's radius for hydrogen and accordingly, $Z_{\text{eff}} \geq 1$, as the case may be. Taking the square of Eq. (15) and substituting into Eq. (12) gives:

$$\boldsymbol{\alpha} = \sqrt[2]{\frac{2n^2 E_{\text{I}} \pi^2 e^4 m_{\text{e}}^2 a^2}{m_{\text{e}} c^2 n^4 h^4 \varepsilon_0^2}} \tag{16}$$

$$= \sqrt[2]{2m_{\text{e}} E_{\text{I}}} \times \frac{\pi a e^2}{nh^2 \varepsilon_0 c} \tag{17}$$

Equating step 14 (Eq. (14)) with step 17 (Eq. (17)), rearranging and simplifying lead to:

$$a = \frac{nh}{2\pi\sqrt{(2m_e E_{\text{I}})}} \left(or \frac{nh}{\pi\sqrt{8m_e E_{\text{I}}}} \right) \tag{18}$$

Equation (18) is exactly the equation derived elsewhere [12]. Since **α** is universal, Eq. (17) can be re-expressed as

$$a = \frac{\boldsymbol{\alpha} nh^2 \varepsilon_0 c}{\pi e^2 \sqrt[2]{2m_e E_{\text{I}}}} \tag{19}$$

$$\cong \frac{1}{137} \frac{nh^2 \varepsilon_0 c}{\pi e^2 \sqrt[2]{2m_e E_{\text{I}}}} \tag{20}$$

$$\cong \frac{1}{137} \frac{nh\varepsilon_0 c}{\pi e^2} \lambda_{\text{dB(e)}} \tag{21}$$

where $\lambda_{\text{dB(e)}}$ is the de Broglie wavelength of the electron in motion with the understanding that E_{I} is ≡ kinetic energy.

2.3.2 Relating fine structure constant to effective nuclear charge

Although equations (9) and (15) are equations for Z_{eff} the equations do not reflect the presence of **α** as a universal constant. The purpose of this subsection is to relate **α** with Z_{eff}. By taking the reciprocal of **α** in Eq. (2) and substituting Eq. (11) into it gives after simplification the solution:

$$\frac{1}{\boldsymbol{\alpha}} = \frac{c}{v} \equiv c \sqrt[2]{\frac{m_e}{2E_{\text{I}}}} \frac{Z_{\text{eff}}}{n} \quad \therefore \tag{22}$$

$$Z_{\text{eff}} = \frac{n}{\boldsymbol{\alpha} c} \sqrt[2]{\frac{2E_{\text{I}}}{m_e}} \tag{23}$$

3. METHODS

The research involves theoretical methods in which data in literature [13] were substituted into derived equations [13]. The average first ionization energies of some elements were arbitrarily selected from each group of elements in the periodic table for the purpose of illustration. However, a supplementary data on the results of illustrative calculation are presented so as to preclude doubt. A step by step approach is adopted in deriving the equations so as to avoid making any error.

4. RESULTS AND DISCUSSION

From the derivational steps so far, there is no where very high level quantum/wave mechanics with its complex mathematical formalism is applicable. It appears that classical approach in the application of kinetic energy to the solution to the question of whether or not fine structure constant is a myth has been the case in this investigation. This is to say that modern approaches such as Heisenberg uncertain principle and Schrödinger formalism that describes an electron in an atom is precluded for reasons which do not arise from recent criticism of such approach [7]. "Despite its wide acceptance, on deeper inspection, the Schrodinger equation solution is plagued with many failings as well as difficulties in terms of a physical interpretation that have caused it to remain controversial since its inception" [7]. The author [7] described negative kinetic energy as nonsensical. Support for Hills' position is already indicated in Eq. (1). The question asked recently is why does radiative collapse of electrons undergoing artificially induced circular motion made possible at extreme voltage in a betatron for instance, does not occur [12].

Equation (12) shows that Z_{eff} and average ionization energy, E_I are related to fine structure constant**,** but E_I is dependent on Z_{eff} and cancels out following substitution leading to Eq. (14). So far it can be seen that Eq. (14) remains what is usually found in standard text books and justify the claim in this research that α is intrinsic to all elements and consequently has an attribute of universality. If the parameters are well-known physical constant, then there is no basis whatsoever to mystify the term fine structure constant. There have been issues concerning the uses of α [3]. In this investigation, it is obvious that it can be adopted for the determination of Bohr's radius of any atom, apart from hydrogenic atoms without approximations that has been bitterly criticized [7]. The results obtained using Eq. (18) which, had been reported in literature [12] is replicated here as shown in Table 1. The result in Table 1 is obtained by using directly the value of 'α'.

As shown in Table 1, the values of Z_{eff} and '*a*' calculated using corresponding equation do not differ widely. The dependence of Z_{eff} and '*a*' on fine structure constant does not lead to results that are widely different from each other. However, the results presented in Table 1 differ from those reported in literature [14-16]. It should be stated however, that an approach different from the approach in this research has been reported in literature [17].

It is not out of place to opine that Bohr's mathematical model for hydrogen (but not to the exclusion of non-hydrogenic atom) shows that '*a*' is directly proportional to n^2 and inversely proportional Z_{eff}. Whereas, as previously observed earlier elsewhere [12], Eq. (19) and Eq. (20) show that '*a*' is directly proportional to *n* and

Table 1. Calculated effective nuclear charge and Bohr's radius of some elements from 1st principle dependent derived equations

Selected elements	*E_I/kJ	Z_{eff} (Eq.(7))	Z_{eff} (Eq. (17))	(*a*/Å) (Eq. (14))	(*a*/Å) (Eq. (15b))	(*a*/Å) (Eq. (12))
$_2$He	2372	1.344155	1.344294	0.393672	0.393631	0.393672
$_6$C	1087	1.819856	1.820044	1.163073	1.162953	1.163073
$_7$N	1402	2.066789	2.067003	1.024113	1.024006	1.024113
$_8$O	1314	2.000874	2.001081	1.057850	1.057741	1.057850
$_{13}$Al	578	1.990569	1.990775	2.392484	2.392237	2.392484
$_{17}$Cl	1251	2.928479	2.928781	1.626239	1.626071	1.626239
$_{11}$Na	496	1.843972	1.844162	2.582689	2.582423	2.582689
$_{20}$Ca	590	2.681502	2.681779	3.157372	3.157046	3.157372
$_{26}$Fe	763	3.049400	3.049715	2.776448	2.776161	2.776448
$_1$H	1312	0.999779	0.999676	0.529328	0.529273	0.529273

*The parameters, E_I, Z_{eff}, and 'a' are average ionization, effective nuclear charge, and Bohr's radius for any element. * signifies values obtained from Kneen et al. [13]. Calculated values were approximated to 6 decimal places in order not to mask slight differences*

inversely proportional to the square root of average ionization energy. The same value of '*a*' is obtainable using Eq. (18), Eq. (19), and Eq. (20) for both hydrogenic and non-hydrogenic atoms (Table 1). Equation (21) shows that *a* is directly proportional to the de Broglie wave length of the electron which, may be within what has been called a fractional quantum energy levels otherwise called inverse quantum state of atom [18,19], ground state energy level or above ground state energy level-$n \geq 1$. On the other hand, Eq. (23) shows that Z_{eff} is directly proportional to principal quantum number and to the square root of averaged ionization energy, not only for hydrogen but for every non-hydrogenic atom. The calculated values (Table 1) of the selected atoms (as in literature [12]) using Eq. (9) and Eq. (23) are the same.

5. CONCLUSION

Several equations have been derived in different ways. Given physical parameter such as radius of any atom (not just hydrogen or any hydrogenic atom), the fine structure constant can be determined. Therefore, the fine structure constant is an intrinsic property of matter. Thus successfully derived equations related the fine structure constant with well known periodic properties of elemental matter, effective nuclear charge, average ionization energy, and Bohr's radius for any atom, other than hydrogenic atoms. This research began in the 1990s but no further action was taken until recently due to believe in scientific community that elementary particle cannot attain let alone exceed the speed of light; but the question is what the situation now is? This issue in addition to the derivation of an equation that can be used to determine the Bohr's radius of any atom with and without fine structure constant will be addressed in the feature.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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APPENDIX A

Why Equation stated in the abstract section is very unambiguous and valid

The presence of the fine structure constant in the equation and elsewhere in the text can be justified on account of the following derivation. Consequently the presence of Z_{eff} is not out of place. To begin with, it should be appreciated that regardless of whom the author of earlier idea might be, such idea is amenable to confirmation and disproof or it can be used innovatively to solve other problem in a more generalizable manner. This is how Bohr's mathematical model is treated in this research and elsewhere [12].

An average ionization energy (or kinetic energy) should be seen as a positive quantity. Negative kinetic energy has been described as nonsensical in literature [7]. The first ionization energy (IE) per mole covers Avogadro's number (N_A) of electrons; the removal of one electron to a position at infinity requires positive IE/N_A – the average ionization energy. The equation of positive average ionization energy in the light of literature information [7] and common sense, IE/N_A (*i.e*. E_I as in the text) is given elsewhere [20] as:

$$E_{\mathrm{I}} = \frac{Z_{\mathrm{eff}}^2 e^4 m_e}{8\varepsilon_0^2 n^2 h^2} \tag{A1}$$

The expression which links effective nuclear charge, Z_{eff} to fine structure constant in abstract section is derived from Eq. (12), and it is similar to Eq. (9). Substitution of Eq. (A1) into Eq. (12) or derived form in the abstract leads to, after simplification,

$$\alpha = \sqrt[2]{\frac{2n^2}{Z_{\mathrm{eff}}^2 m_{\mathrm{e}} c^2} \frac{Z_{\mathrm{eff}}^2 e^4 m_e}{8\varepsilon_0^2 n^2 h^2}}$$

$$= \frac{\boldsymbol{e^2}}{\boldsymbol{2hc\varepsilon_0}} \tag{A.2}$$

Equation (A.2) is as in most standard text books and as stated herein.

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