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The Effect of Anisotropy on the Structure Optimization Using Golden-Section Search Algorithm Based on BEM

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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Abstract

Aims: A shape optimization technique is developed, using the boundary element method, for two-dimensional anisotropic structures to study the effects of anisotropy on the displacements and stresses, then minimize weight while satisfying certain constraints upon stresses and geometry.
Study Design: Original Research Paper.
Place and Duration of Study: Jamoum University College, Mathematics Department, between June 2016 and July 2017.
Methodology: The shape design sensitivity analysis of a two-dimensional anisotropic structure using a singular formulation of the boundary element method is investigated to study the effects of anisotropy on the displacements and stresses. An Implicit differentiation technique of the discretized boundary integral equations is performed to produce terms that contain derivatives of the fundamental solutions employed in the analysis. This technique allows the coupling between optimization technique and numerical boundary element method (BEM) to form an optimum shape design algorithm that yields shape design sensitivities of the displacement and stress fields for anisotropic materials with very high accuracy. The fundamental solutions of displacements and tractions in terms of complex variables employed in the analysis. The feasible direction method was developed and implemented for use with the golden-section search algorithm based on BEM as a numerical optimization technique for minimizing weight while satisfying all of the constraints.

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Results: The proposed method has been verified by using the two-dimensional plate with an elliptical hole as the numerical example. The numerical results show that the proposed method is suitable and effective tool for the computer implementation of the solution.
Conclusion: From the research that has been performed, it is possible to conclude that the optimal shape of the two-dimensional plate with an elliptical hole is crucial when elastic field is sensitive to boundary shape. Also from this knowledge of the effects of anisotropy on the displacements and stresses, we can design various anisotropic structures to meet specific engineering requirements and utilize within which to place new information can be more effective.

Keywords: Shape optimization; design sensitivity; implicit differentiation method; anisotropic structures; boundary element method.

2010 Mathematics subject classification: 65M38 - 65K05 - 74B05 - 74E05 -74F05 - 74H05 - 74H15 - 74S20 - 90C31.

1 Introduction

The rapid development of composite materials following wide varieties of techniques and the design and manufacturing technologies is one of the most significant achievements in the field of materials engineering and science. Because of their high stiffness and high strength properties, composites are the most commonly used in mechanical engineering and aerospace applications [1-5].

In recent years, the scientific research in the field of optimization algorithms has become a rapidly developing area of research in computational optimization techniques [6-8].

Numerical techniques are also increasingly used for analysis of structural engineering, among which the boundary element technique [9-19], which offers a clear advantage over other methods, and is applicable to a wide range of structural engineering problems. The study of anisotropic structures is very complex and is still not well understood, and as a result, more sophisticated strategies for optimal design of anisotropic structure are in demand [20-25].

2 Formulation of the Problem

The equilibrium equation for anisotropic elasticity

$$\sigma_{ij,j} + X_j = 0 \tag{1}$$

where there are 21 independent material elastic constants C_{ijkl} because $C_{ijkl} = C_{jikl} = C_{ijlk} = C_{klij}$. Due to the symmetry of the stress and strain tensors, and assuming that the material is symmetric with respect to the z-direction which is perpendicular to $x - y$ plane, the two-dimensional stress-strain relations for plane-stress anisotropic elasticity are

$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{bmatrix} = \begin{bmatrix} d_{11} & d_{12} & d_{16} \\ d_{12} & d_{22} & d_{26} \\ d_{16} & d_{26} & d_{66} \end{bmatrix} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} \tag{2}$$

where σ_{ij} and ε_{ij} $(i,j = x,y)$ are the stress and strain components, respectively, d_{pq} are the elastic compliances coefficients, which can be written in terms of elastic constants as follows

$$d_{11} = \frac{1}{E_1}, d_{22} = \frac{1}{E_2}, a_{12} = -\frac{\nu_{12}}{E_1} = -\frac{\nu_{21}}{E_2}$$
$$d_{66} = \frac{1}{\mu_{12}}, \qquad d_{16} = \frac{\eta_{12,1}}{E_1} = \frac{\eta_{1,12}}{\mu_{12}}, d_{26} = \frac{\eta_{12,2}}{E_2} = \frac{\eta_{2,12}}{\mu_{12}} \tag{3}$$

The coefficients of mutual influence of the first and second kinds, respectively

$$c_{pq} = d_{pq} - \frac{d_{p3}d_{q3}}{d_{33}}, \qquad p, q = 1, 2, 6$$

$$d_{p3} = -\frac{\nu_{p3}}{E_p}, d_{33} = \frac{1}{E_3}, d_{63} = \frac{\eta_{12,3}}{E_3} = \frac{\eta_{3,12}}{\mu_{12}} \tag{4}$$

The strain compatibility equation is

$$\frac{\partial^2 \varepsilon_{11}}{\partial x_2^2} + \frac{\partial^2 \varepsilon_{22}}{\partial x_1^2} = 2\frac{\partial^2 \varepsilon_{12}}{\partial x_1 \partial x_2} \tag{5}$$

The equilibrium equation is automatically satisfied by writing the stresses in terms of derivatives of the Airy stress function $\phi(\mathrm{x}_1, \mathrm{x}_2)$ proposed by Airy [26] as

$$\sigma_{11} = \frac{\partial^2 \phi}{\partial x_2^2}, \sigma_{22} = \frac{\partial^2 \phi}{\partial x_1^2}, \sigma_{12} = \frac{\partial^2 \phi}{\partial x_1 \partial x_2} \tag{6}$$

Now by combining equations (2), (5) and (6), the governing equation for the two-dimensional anisotropic elasticity can be obtained as

$$d_{22}\frac{\partial^4 \phi}{\partial x_1^4} - 2d_{26}\frac{\partial^4 \phi}{\partial x_1^3 \partial x_2} + (2d_{12} + d_{66})\frac{\partial^4 \phi}{\partial x_1^2 \partial x_2^2} - 2d_{16}\frac{\partial^4 \phi}{\partial x_1 \partial x_2^3} + d_{11}\frac{\partial^4 \phi}{\partial x_2^4} = 0 \tag{7}$$

It is convenient to define the operator f_s $(s = 1, 4)$ as follows

$$f_s = \frac{\partial}{\partial x_2} - G_s\frac{\partial}{\partial x_1} \tag{8}$$

equation (7) may be reexpressed in the following form

$$f_1 f_2 f_3 f_4(\phi) = 0 \tag{9}$$

where G_s are the four roots of the following characteristic equation

$$[d_{22} - 2Gd_{26} + (2d_{12} + d_{66})G^2 - 2d_{16}G^3 + d_{11}G^4]\frac{d^4 \phi}{dz^4} = 0 \tag{10}$$

Lehknitskii [27] has proved that, for an anisotropic elastic material, these roots are distinct and should be purely imaginary or complex and they can be denoted by

$$G_1 = a_1 + ib_1, G_2 = a_2 + ib_2, G_3 = \overline{G_1}, G_4 = \overline{G_2} \tag{11}$$

where a_j and b_j $(j = 1, 2)$ are real constants, $i = \sqrt{-1}$ and the overbar is complex conjugate. Thus, the stresses and displacements in an anisotropic elastic body may be expressed in terms of the complex coordinates $z_j = x_1 + G_j x_2$ and their complex conjugates

Also, the strains can be written in terms of the stresses in non-principal coordinate system of laminae as [28]

$$\begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{xy}\end{bmatrix}=\begin{bmatrix}\bar{d}_{11} & \bar{d}_{12} & \bar{d}_{16}\\ \bar{d}_{12} & \bar{d}_{22} & \bar{d}_{26}\\ \bar{d}_{16} & \bar{d}_{26} & \bar{d}_{66}\end{bmatrix}\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{bmatrix} \tag{12}$$

Where the transformed compliances are

$$\bar{d}_{11}=\vartheta_1+\vartheta_2\cos 2\varrho+\vartheta_3\cos 4\varrho \tag{13a}$$

$$\bar{d}_{12}=\vartheta_4-\vartheta_3\cos 4\varrho \tag{13b}$$

$$\bar{d}_{22}=\vartheta_1-\vartheta_2\cos 2\varrho+\vartheta_3\cos 4\varrho \tag{13c}$$

$$\bar{d}_{16}=\vartheta_2\sin 2\varrho+2\vartheta_3\sin 4\varrho \tag{13d}$$

$$\bar{d}_{26}=\vartheta_2\sin 2\varrho-2\vartheta_3\sin 4\varrho \tag{13e}$$

$$\bar{d}_{66}=2(\vartheta_1-\vartheta_4)-4\vartheta_3\cos 4\varrho \tag{13f}$$

in which the invariants $(\vartheta_1,\vartheta_2,\vartheta_3,\vartheta_4)$ are

$$\vartheta_1=\frac{1}{8}(3d_{11}+3d_{22}+2d_{12}+d_{66}) \tag{14a}$$

$$\vartheta_2=\frac{1}{2}(d_{11}-d_{22}) \tag{14b}$$

$$\vartheta_3=\frac{1}{8}(d_{11}+d_{22}-2d_{12}-d_{66}) \tag{14c}$$

$$\vartheta_4=\frac{1}{8}(d_{11}+d_{22}+6d_{12}-d_{66}) \tag{14d}$$

3 Numerical Implementation

According to the Betti's reciprocal theorem, we assume that σ_{ij}, e_{ij} and σ_{ij}^*, e_{ij}^* represent two different types of stresses and strains which satisfy equilibrium, compatibility and Hooke's law. Hence

$$\int_R \sigma_{ij}e_{ij}^*\,dR=\int_R \sigma_{ij}^*e_{ij}\,dR \tag{15}$$

the unstarred quantities are supposed to represent the unknown solution, and the starred quantities correspond to a singular fundamental solution to Navier's equation, associated with a point load in an infinite domain, which after performing integration by parts twice and applying the divergence theorem, leads to the following somigliana displacement identity (Cruse [29])

$$u_i(\xi)=\int_s t_j(\eta)U_{ij}(\xi,\eta)ds(\eta)-\int_s u_j(\eta)T_{ij}(\xi,\eta)ds(\eta) \tag{16}$$

The displacements and tractions fundamental solutions are respectively as follows:

$$U_{ij}(\xi,\eta) = \frac{1}{8\pi\mu(1-\nu')} \times \left\{(3-4\nu')\delta_{ij} \ln\left(\frac{1}{r}\right) + r_{,i}r_{,j}\right\} \tag{17}$$

$$T_{ij}(\xi,\eta) = -\frac{1}{4\pi(1-\nu')}\left(\frac{1}{r}\right) \times \left\{\frac{\partial r}{\partial n}(1-2\nu')\delta_{ij} + 2r_{,i}r_{,j}\right\}$$

$$-(1-2\nu')\left(n_j r_{,i} - n_i r_{,j}\right) \tag{18}$$

where $r = \|\xi - \eta\|$ is the Euclidean distance between the load point $\xi = (\xi_1, \xi_2)$ and the field point $\eta = (x_1, x_2)$

$$r = \{(x_i - \xi_i)(x_i - \xi_i)\}^{1/2} \tag{19}$$

The derivatives of $r(\xi,\eta)$ with respect to the field point coordinates and the unit normal vector at the field point are as follows

$$r_{,i} = \frac{\partial r}{\partial x_i} = \frac{1}{r}(x_i - \xi_i) = -\frac{\partial r}{\partial \xi_i} \tag{20}$$

$$\frac{\partial r}{\partial n} = \frac{\partial r}{\partial x_i} n_i \tag{21}$$

The boundary integral equation in the

$$u_m(\xi) = \int_s t_m(\eta) U_{mn}(\xi,\eta) ds(\eta) - \int_s u_m(\eta) T_{mn}(\xi,\eta) ds(\eta) \tag{22}$$

The boundary integral equation resulting from the direct boundary element formulation for anisotropic structures, may be written as

$$D_{mn}u_m(\xi) + \int_s T_{mn}(\xi,\eta) u_m(\eta) ds(\eta) = \int_s U_{mn}(\xi,\eta) t_m(\eta) ds(\eta) \tag{23}$$

where $\xi(\xi_1, \xi_2)$ and $\eta = (x, y)$ are the load and field points, respectively. $U_{mn}(\xi,\eta)$ and $T_{mn}(\xi,\eta)$ are the fundamental solutions which represent the displacements and tractions, respectively,

The coefficient D_{mn} depends on the local geometry of the boundary at ξ, which lies on the smooth surface or a sharp corner, $m, n = 1, 2$.

4 Shape Design Sensitivity Analysis of 2D Anisotropic Structures

Implicit differentiation of the boundary integral equation (23) with respect to the design variable x_h yields the following equation [25]

$$D_{mn}\frac{\partial u_m(\xi)}{\partial x_h} + \frac{\partial D_{mn}}{\partial x_h} u_m(\xi)$$

$$+ \int_s \left(\frac{\partial T_{mn}(\xi,\eta)}{\partial x_h} u_m(\eta) + T_{mn}(\xi,\eta)\frac{\partial u_m(\eta)}{\partial x_h}\right) ds(\eta)$$

$$+\int_s T_{mn}(\xi,\eta)u_m(\eta)\frac{\partial(ds(\eta))}{\partial x_h}$$

$$=\int_s \left(\frac{\partial U_{mn}(\xi,\eta)}{\partial x_h}t_m(\eta)+U_{mn}(\xi,\eta)\frac{\partial t_m(\eta)}{\partial x_h}\right)ds(\eta)$$

$$+\int_s U_{mn}(\xi,\eta)t_m(\eta)\frac{\partial(ds(\eta))}{\partial x_h} \tag{24}$$

$\bar{m}_{1n}$ is the unit vector in the tangential direction to the surface, and $\bar{m}_{2n}$ is the unit vector in the normal direction to the surface, let $u_m, t_m, \varepsilon_{mn}$ and σ_{mn} be the displacements, tractions, strains and stresses, respectively.

The tangential displacement is

$$\bar{u}_m(\xi)=N^c(\xi)u_m^c\bar{m}_{1n}$$

and the tangential strain is

$$\bar{\varepsilon}_{xx}(\xi)=\frac{dN^c(\xi)}{d\xi}u_m^c\bar{m}_{1n}\frac{1}{J(\xi)}$$

The gradients of the tangential strain may be written as follows

$$\frac{\partial\bar{\varepsilon}_{xx}}{\partial x_h}=\frac{dN^c(\xi)}{d\xi}\frac{\partial u_m^c}{\partial x_h}\bar{m}_{1n}\frac{1}{J(\xi)}+\frac{dN^c(\xi)}{d\xi}u_m^c\frac{\partial\bar{m}_{1n}}{\partial x_h}\frac{1}{J(\xi)}$$

$$+\frac{dN^c(\xi)}{d\xi}u_m^c\bar{m}_{1n}\frac{-1}{[J(\xi)]^2}\frac{\partial[J(\xi)]}{\partial x_h} \tag{25}$$

The fundamental solutions can be written in a more concise form by introducing the following complex variables:

$$z_1=(x-\xi_1)+G_1(y-\xi_2) \tag{26a}$$

$$z_2=(x-\xi_1)+G_2(y-\xi_2) \tag{26b}$$

In terms of above complex variables the fundamental solutions for displacements and tractions, respectively, may be written in the following form:

$$U_{jk}=2Re\left[r_{1m}A_{j1}\ln(z_1)+r_{2m}A_{j2}\ln(z_2)\right], \tag{27}$$

$$T_{m1}=2n_1Re\left[\frac{G_1^2A_{m1}}{z_1}+\frac{G_2^2A_{m2}}{z_2}\right]-2n_2Re\left[\frac{G_1A_{m1}}{z_1}+\frac{G_2A_{m2}}{z_2}\right] \tag{28}$$

$$T_{m2}=-2n_1Re\left[\frac{G_1A_{m1}}{z_1}+\frac{G_2A_{m2}}{z_2}\right]+2n_2Re\left[\frac{A_{m1}}{z_1}+\frac{A_{m2}}{z_2}\right] \tag{29}$$

According to $x-y$ coordinate system, the n_m are the outward unit normal components at η and the constants r_{nm} are

$$r_{1m} = d_{11}G_m^2 + d_{12} - d_{16}G_m, \qquad r_{2m} = d_{12}G_m + d_{22}/G_m - d_{26} \tag{30}$$

and A_{mn} are complex constants that can be determined from the following matrix equations

$$[\mathrm{Im}\{B_1\} \quad \mathrm{Re}\{B_1\} \quad \mathrm{Im}\{B_2\} \quad \mathrm{Re}\{B_2\}] = \begin{bmatrix} \mathrm{Re}\{A_{m1}\} \\ \mathrm{Im}\{A_{m1}\} \\ \mathrm{Re}\{A_{m2}\} \\ \mathrm{Im}\{A_{m2}\} \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{4\pi}\delta_{m2} \\ \frac{1}{4\pi}\delta_{m1} \\ 0 \\ 0 \end{bmatrix}; m = 1,2 \tag{31a}$$

$$\{B_n\} = \{i \quad \mu_n \quad r_{1n} \quad r_{2n}\}^T; \quad n = 1,2 \tag{31b}$$

For further details, we refer the reader to Cruse [29]

According to the numerical procedure for the boundary element implementation of Fahmy [30-33], we obtain from equation (23) the following system of linear algebraic equations which has to be solved using any of the standard matrix reduction techniques to obtain the unknown displacements and tractions at the boundary as:

$$\mathbb{A}\mathbb{U} = \mathbb{B} \tag{32}$$

where $\mathbb{A}$ and $\mathbb{B}$ are the matrices that contain evaluated integrals of the fundamental displacement and traction kernels, respectively

Let m_{1n} is the tangential unit vector to the surface and m_{2n} is the normal unit vector to the surface. Let u_m, σ_{mn}, ε_{mn} and t_m are, respectively, the displacements, stresses, strains and tractions in the local coordinates.

The tangential displacement is

$$\bar{u}_m(\xi) = N^c(\xi)u_m^c m_{1n} \tag{33}$$

where $N^c(\xi)$ $(c = 1,2,3)$

and tangential strain is

$$\bar{\varepsilon}_{xx}(\xi) = \frac{dN^c(\xi)}{d\xi}u_m^c m_{1n}\frac{1}{J(\xi)} \tag{34}$$

In the local coordinates the stress components can be calculated by using the constitutive equation (2)

The derivatives with respect to a design variable x_h $(h = 1,2)$ for anisotropic materials will be as follows:

$$\frac{\partial U_{mn}}{\partial x_h} = 2\frac{\partial}{\partial x_h}[Re(r_{n1}A_{m1}\ln(z_1) + r_{n2}A_{m2}\ln(z_2))] \tag{35}$$

$$\frac{\partial T_{m1}}{\partial x_h} = 2n_1 \frac{\partial}{\partial x_h}\left[\mathrm{Re}\left(\frac{G_1^2 A_{m1}}{z_1} + \frac{G_2^2 A_{m2}}{z_2}\right)\right] + 2Re\left(\frac{G_1^2 A_{m1}}{z_1} + \frac{G_2^2 A_{m2}}{z_2}\right)\frac{\partial n_1}{\partial x_h}$$
$$-2n_2 \frac{\partial}{\partial x_h}\left[\mathrm{Re}\left(\frac{G_1 A_{m1}}{z_1} + \frac{G_2 A_{m2}}{z_2}\right)\right] - 2\mathrm{Re}\left(\frac{G_1 A_{m1}}{z_1} + \frac{G_2 A_{m2}}{z_2}\right)\frac{\partial n_2}{\partial x_h} \quad (36)$$

$$\frac{\partial T_{m2}}{\partial x_h} = -2n_1 \frac{\partial}{\partial x_h}\left[\mathrm{Re}\left(\frac{G_1 A_{m1}}{z_1} + \frac{G_2 A_{m2}}{z_2}\right)\right] - 2\mathrm{Re}\left(\frac{G_1 A_{m1}}{z_1} + \frac{G_2 A_{m2}}{z_2}\right)\frac{\partial n_1}{\partial x_h}$$
$$+2n_2 \frac{\partial}{\partial x_h}\left[\mathrm{Re}\left(\frac{A_{m1}}{z_1} + \frac{A_{m2}}{z_2}\right)\right] + 2\mathrm{Re}\left(\frac{A_{m1}}{z_1} + \frac{A_{m2}}{z_2}\right)\frac{\partial n_2}{\partial x_h} \quad (37)$$

To compute the previous derivatives, the complex values $\ln(z_j)$ and $\frac{1}{z_j}$ may be written as follows

$$\ln(z_j) = \ln|z_j| + i\arg(z_j), \qquad \frac{1}{z_j} = \frac{\bar{z}_j}{|z_j|^2} \quad (38)$$

It is convenient to introduce the following real functions

$$\lambda_1 = (x_1 - \xi_1) + \propto_1 (x_2 - \xi_2) \quad (39)$$

$$\lambda_2 = (x_1 - \xi_1) + \propto_2 (x_2 - \xi_2) \quad (40)$$

$$\Psi_1 = -\beta_1 \xi_2 + \beta_1 x_2 \quad (41)$$

$$\Psi_2 = -\beta_2 \xi_2 + \beta_2 x_2 \quad (42)$$

The complex coordinates and their complex conjugates can be written in the following form

$$z_j = \lambda_j + i\Psi_j, \;\; \bar{z}_j = \lambda_j - i\Psi_j \,, j = 1, 2 \quad (43)$$

By substituting from equations (38-43) into equations (35-37) we obtain

$$\frac{\partial U_{mn}}{\partial x_h} = 2\mathrm{Re}(r_{n1} A_{m1}) \frac{\partial}{\partial x_h}[\ln|z_1|] + 2\mathrm{Re}(r_{n1} A_{m1} i) \frac{\partial}{\partial x_h}[\arg(z_1)]$$
$$+2\mathrm{Re}(r_{n2} A_{m2}) \frac{\partial}{\partial x_h}[\ln|z_2|] + 2\mathrm{Re}(r_{n2} A_{m2} i) \frac{\partial}{\partial x_h}[\arg(z_2)] \quad (44)$$

$$\frac{\partial T_{m1}}{\partial x_h} = 2\mathrm{Re}\left[\frac{G_1^2 A_{m1}}{z_1} + \frac{G_2^2 A_{m2}}{z_2}\right]\frac{\partial n_1}{\partial x_h} - 2\mathrm{Re}\left(\frac{G_1 A_{m1}}{z_1} + \frac{G_2 A_{m2}}{z_2}\right)\frac{\partial n_2}{\partial x_h}$$
$$+2n_1 Re(G_1^2 A_{m1}) \frac{\partial}{\partial x_h}\left(\frac{\lambda_1}{|z_1|^2}\right) + 2n_1 Re(-G_1^2 A_{m1} i) \frac{\partial}{\partial x_h}\left(\frac{\Psi_1}{|z_1|^2}\right)$$
$$+2n_1 Re(G_2^2 A_{m2}) \frac{\partial}{\partial x_h}\left(\frac{\lambda_2}{|z_2|^2}\right) + 2n_1 Re(-G_2^2 A_{m2} i) \frac{\partial}{\partial x_h}\left(\frac{\Psi_2}{|z_2|^2}\right)$$
$$-2n_2 \mathrm{Re}(G_1 A_{m1}) \frac{\partial}{\partial x_h}\left(\frac{\lambda_1}{|z_1|^2}\right) - 2n_2 \mathrm{Re}(-G_1 A_{m1} i) \frac{\partial}{\partial x_h}\left(\frac{\Psi_1}{|z_1|^2}\right)$$
$$-2n_2 Re(G_2 A_{m2}) \frac{\partial}{\partial x_h}\left(\frac{\lambda_2}{|z_2|^2}\right) - 2n_2 \mathrm{Re}(-G_2 A_{m2} i) \frac{\partial}{\partial x_h}\left(\frac{\Psi_2}{|z_2|^2}\right) \quad (45)$$

$$\frac{\partial T_{m2}}{\partial x_h} = -2\mathrm{Re}\left[\frac{G_1 A_{m1}}{z_1} + \frac{G_2 A_{m2}}{z_2}\right]\frac{\partial n_1}{\partial x_h} + 2\mathrm{Re}\left(\frac{A_{m1}}{z_1} + \frac{A_{m2}}{z_2}\right)\frac{\partial n_2}{\partial x_h}$$

$$-2n_1 Re(G_1 A_{m1})\frac{\partial}{\partial x_h}\left(\frac{\lambda_1}{|z_1|^2}\right) - 2n_1 Re(-G_1 A_{m1} i)\frac{\partial}{\partial x_h}\left(\frac{\Psi_1}{|z_1|^2}\right)$$

$$-2n_1 Re(G_2 A_{m2})\frac{\partial}{\partial x_h}\left(\frac{\lambda_2}{|z_2|^2}\right) - 2n_1 \mathrm{Re}(-G_2 A_{m2} i)\frac{\partial}{\partial x_h}\left(\frac{\Psi_2}{|z_2|^2}\right)$$

$$+2n_2 Re(A_{m1})\frac{\partial}{\partial x_h}\left(\frac{\lambda_1}{|z_1|^2}\right) + 2n_2 \mathrm{Re}(-A_{m1} i)\frac{\partial}{\partial x_h}\left(\frac{\Psi_1}{|z_1|^2}\right)$$

$$+2n_2 \mathrm{Re}[A_{m2}]\frac{\partial}{\partial x_h}\left(\frac{\lambda_2}{|z_2|^2}\right) + 2n_2 \mathrm{Re}(-A_{m2} i)\frac{\partial}{\partial x_h}\left(\frac{\Psi_2}{|z_2|^2}\right) \tag{46}$$

Thus, the design sensitivity analysis is performed by implicit differentiation of equation (32) that describes the structural response with respect to the design variables x_h which are the coordinates of several nodes on the movable part of the boundary

$$\frac{\partial \mathbb{A}}{\partial x_h} U + A\frac{\partial U}{\partial x_h} = \frac{\partial \mathbb{B}}{\partial x_h} \tag{47}$$

Which may be expressed in the following form

$$\mathbb{A}\frac{\partial U}{\partial x_h} = \left(\frac{\partial \mathbb{B}}{\partial x_h} - \frac{\partial \mathbb{A}}{\partial x_h} U\right) \tag{48}$$

This is a set of linear algebraic equations to compute the unknown displacement and traction gradients. After obtaining the displacement sensitivity, the stress sensitivity can be obtained.

The tangential strain's sensitivity can be obtained by differentiating equation (34) as follows

$$\frac{\partial \bar{\varepsilon}_{xx}}{\partial x_h} = \frac{dN^c(\xi)}{d\xi}\frac{\partial u_n^c}{\partial x_h} m_{1n}\frac{1}{J(\xi)} + \frac{dN^c(\xi)}{d\xi} u_n^c \frac{\partial m_{1n}}{\partial x_h}\frac{1}{J(\xi)}$$

$$+\frac{dN^c(\xi)}{d\xi} u_n^c m_{1n}\frac{-1}{[J(\xi)]^2}\frac{\partial [J(\xi)]}{\partial x_h} \tag{49}$$

The elastic compliance can be computed as the strain energy of the structure

$$E_s = \frac{1}{2}\int_s t_m u_m ds \tag{50}$$

If the structure's boundary is discretized into Q quadratic boundary elements. Then, the elastic compliance can be computed as

$$E_s = \frac{1}{2}\sum_{b=1}^{Q}\int_{s_b} [t_m^c(\xi)N^c(\xi)]\,[u_m^d(\xi)N^d(\xi)]J(\xi)d\xi \tag{51}$$

After the displacements, stresses, tractions and their sensitivities are evaluated, the sensitivities of elastic compliance with respect to the boundary point coordinates x_h can also be calculated by implicit differentiation method as follows

$$\frac{\partial E_s}{\partial x_h} = \frac{1}{2}\sum_{b=1}^{Q}\int_{s_b}\left[\frac{dt_m^c}{dx_h}N^c(\xi) + t_m^c(\xi)\frac{dN^c(\xi)}{dx_h}\right]\left[[u_m^d(\xi)N^d(\xi)]J(\xi)d\xi\right]$$

$$+\frac{1}{2}\sum_{b=1}^{Q}\int_{s_b}[t_m^c(\xi)N^c(\xi)]\left[\frac{du_m^d(\xi)}{dx_h}N^d(\xi) + u_m^d(\xi)\frac{dN^d(\xi)}{dx_h}\right]$$

$$\times J(\xi)d\xi + \frac{1}{2}\sum_{\mathrm{b}=1}^{\mathrm{Q}}\int_{\mathrm{s_b}}[t_m^c(\xi)N^c(\xi)]\,[u_m^d(\xi)N^d(\xi)]\frac{\partial J(\xi)}{\partial x_h}d\xi \tag{52}$$

5 Numerical Shape Optimization of 2D Anisotropic Structures

let R be a closed bounded plane region whose boundary c consisting of a finite number of smooth curves and assuming that m and w are continuous functions and have continuous partial derivatives with respect to x_1 and x_2

$$\iint_R \left(\frac{\partial w}{\partial x_1} - \frac{\partial m}{\partial x_2}\right)dx_1dx_2 = \int_c (m\,dx_1 + w\,dx_2) \tag{53}$$

By using the Green's theorem, the area $\bar{A}$ of the domain R $\left(\bar{A} = \iint_R dx_1dx_2\right)$ can be written in terms of a line integral over the boundary

$$\bar{A} = \frac{1}{2}\int_c (x_1\,dx_2 - x_2\,dx_1) \tag{54}$$

If the boundary of the structure is discretized into Q quadratic isoparametric boundary elements, and the coordinates at nodal points can be expressed as

$$x_m(\xi) = N^c(\xi)x_m^c \tag{55}$$

where $N^c(\xi)$ quadratic shape function corresponding to the cth quadrilateral element's node number, and ξ is the intrinsic coordinate for the element. Therefore, the area of the domain can be calculated as follows

$$\bar{A} = \frac{1}{2}\sum_{b=1}^{Q}\int_{-1}^{1}[x_1(\xi)n_1 + x_2(\xi)n_2]J(\xi)d\xi \tag{56}$$

$J(\xi)$ is the Jacobian matrix of the transformation and n_1 and n_2 are direction cosines of the unit normal vector to the surface of the structure which may be written as

$$n_1 = \frac{dx_2}{d\bar{A}} = \frac{dx_2/d\xi}{d\bar{A}/d\xi} = \frac{dx_2/d\xi}{J(\xi)} \tag{57a}$$

$$n_2 = -\frac{dx_1}{d\bar{A}} = -\frac{dx_1/d\xi}{d\bar{A}/d\xi} = -\frac{dx_1/d\xi}{J(\xi)} \tag{57b}$$

Substitution of equations (57) into equation (56) yields

$$\bar{A} = \frac{1}{2}\sum_{b=1}^{Q}\int_{-1}^{1}\left[x_1(\xi)\frac{dx_2}{d\xi} - x_2(\xi)\frac{dx_1}{d\xi}\right]d\xi \tag{58}$$

The weight derivative can be calculated by differentiating (56) with respect to the design variable based on the consideration that, if x_h is the x_1 coordinate of a movable node, then

$$\frac{\partial}{\partial x_h}\left(\frac{dx_2(\xi)}{d\xi}\right) = 0 \tag{59a}$$

and

$$\frac{\partial}{\partial x_h}\left(x_2(\xi)\right) = 0 \tag{59b}$$

Therefore

$$\frac{\partial \bar{A}}{\partial x_h} = \frac{1}{2}\sum_{b=1}^{Q}\int_{-1}^{1}\left[\frac{\partial x_1(\xi)}{\partial x_h}\frac{dx_2}{d\xi} - x_2(\xi)\frac{\partial}{\partial x_h}\left(\frac{dx_1}{d\xi}\right)\right]d\xi \tag{60}$$

If x_h is the x_2 coordinate of a movable node, then

$$\frac{\partial}{\partial x_h}\left(\frac{dx_1(\xi)}{d\xi}\right) = 0 \tag{61a}$$

and

$$\frac{\partial}{\partial x_h}\left(x_1(\xi)\right) = 0 \tag{61b}$$

Therefore

$$\frac{\partial \bar{A}}{\partial x_h} = \frac{1}{2}\sum_{b=1}^{Q}\int_{-1}^{1}\left[x_1(\xi)\frac{\partial}{\partial x_h}\left(\frac{dx_2}{d\xi}\right) - \frac{\partial x_2(\xi)}{\partial x_h}\left(\frac{dx_1}{d\xi}\right)\right]d\xi \tag{62}$$

where weight minimization is equivalent to area minimization.

The general problem that we discuss in the present paper is the minimization of structural weight which must satisfy constraints on stresses and geometry. Since both stress and weight constraints are non-linear functions of the design variables, then the feasible direction approach has been employed as the computational optimization technique. This method determines a usable-feasible direction where the design point can be moved in the design space.

Assuming the weight as the objective function $\bar{A}(x)$ that we want to minimize Subject to constraint function $\chi_i(x) \le k_i,\ i = 1, \dots, M$

According to iterative unconstrained optimization approach, the design variable is updated during the iteration process to find the optimum result as follows

$$x_{h+1} = x_h + s_h d_h \tag{63}$$

where the line step parameter s_h determines the amount of change in x to find the minimum design point along the search direction d_h.

The iteration process must satisfy the following condition

$$\bar{A}(x_{h+1}) - \bar{A}(x_h) \le \varepsilon \tag{64}$$

where ε is the predefined tolerance.

The search direction can be defined as

$$d_h = -H^h \nabla \bar{A}(x_h) \tag{65}$$

where H^h is the $h-$ th approximation of the inverse Hessian matrix, which can be given by

$$H^{h+1} = \left[I - \frac{P^h Q^h}{(P^h)^T Q^h}\right] H^h \left[I - \frac{Q^h (P^h)^T}{(P^h)^T Q^h}\right] + \frac{P^h (Q^h)^T}{(P^h)^T Q^h} \tag{66}$$

where I denote the identity matrix and $(\,)^T$ denote the transpose of the matrix

$$P^h = x_{h+1} - x_h \tag{67}$$

and

$$Q^k = \nabla \bar{A}(x_{h+1}) - \nabla \bar{A}(x_h) \tag{68}$$

$$H^0 = I \tag{69}$$

6 Numerical Results and Discussion

The two-dimensional anisotropic plate with an elliptic hole was used as the numerical example in order to verify the formulation and the implementation of BEM presented in this paper, the physical data of the material of the considered plate for orthotropic (see [1]) and isotropic (see [2]) and for anisotropic is given as follows:

$$\frac{E_1}{E_2} = 13.36, \qquad \frac{\mu_{12}}{E_2} = 0.58 \qquad \nu_{12} = 0.295$$

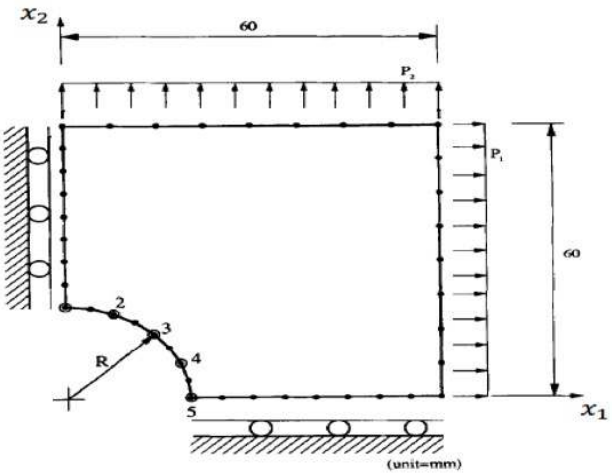

Fig. 1. Boundary element model and design variables of the plate with elliptical hole.

According to the symmetry, only one quarter of the considered two-dimensional plate and the dimensions and notation are given in Fig. 1 where the plate was modeled with linear elements. The design boundary is controlled by five master nodes as the design variables. In order to control the positions of the master nodes which control the shape of the structure, a five-node cubic spline curve fitting technique have been developed and successfully used [34]. The optimum shape of the deterministic design is shown in Fig. 2.

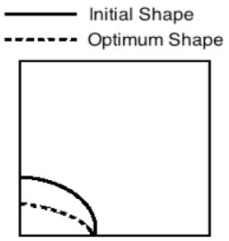

Fig. 2. Optimum shape of infinite anisotropic plate with elliptical hole.

The variations of the displacement components u_1 and u_2 with x coordinate are plotted in Figs. 3 and 4 to show the effects of anisotropy which are very pronounced on the figures

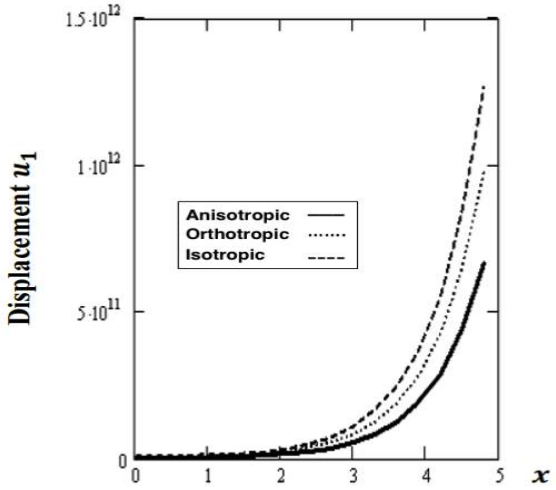

Fig. 3. Variation of the displacement u_1 with x coordinate for Anisotropic, Orthotropic and Isotropic.

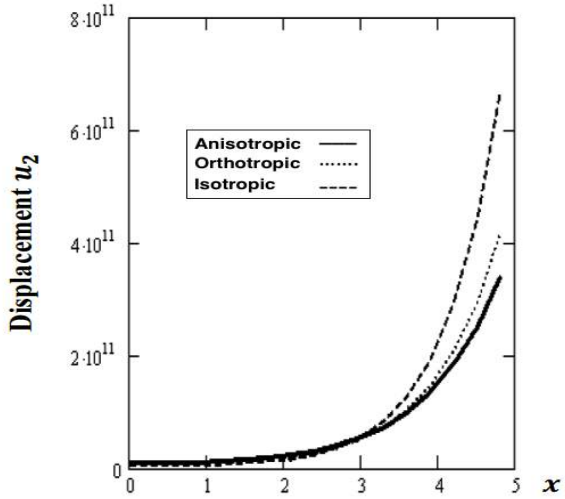

Fig. 4. Variation of the displacement u_2 with x coordinate for Anisotropic, Orthotropic and Isotropic.

The variations of the stress components σ_{11}, σ_{12} and σ_{22} are plotted in Figs. 5, 6 and 7 to show the effects of anisotropy which are very pronounced on the figures

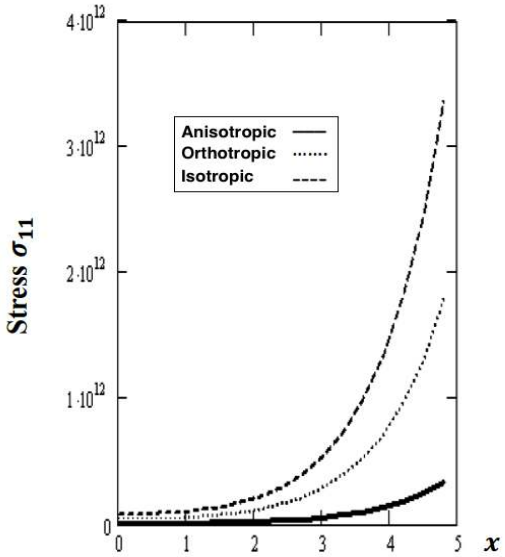

Fig. 5. Variation of the stress σ_{11} with x coordinate for Anisotropic, Orthotropic and Isotropic.

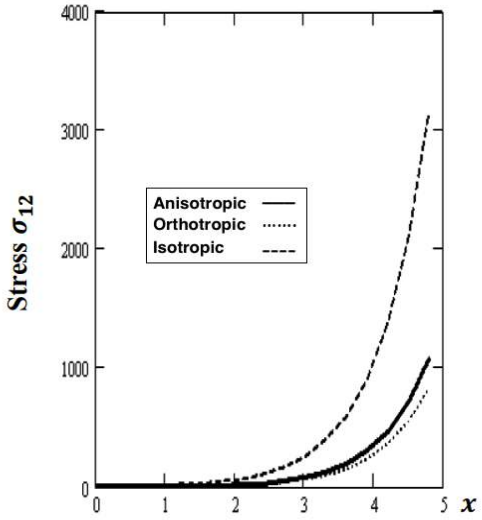

Fig. 6. Variation of the stress σ_{12} with x coordinate for Anisotropic, Orthotropic and Isotropic.

The displacement sensitivities are plotted in Figs. 8 and 9 to verify the formulation and the implementation of BEM. These results obtained with the BEM have been compared graphically with those obtained using the analytical solution of [35] and finite element method of [36]. It can be seen from these figures that the BEM results are in very good agreement with the analytical results and FEM, thus confirming the accuracy of the BEM. Our results thus confirm that our method is strong and efficient.

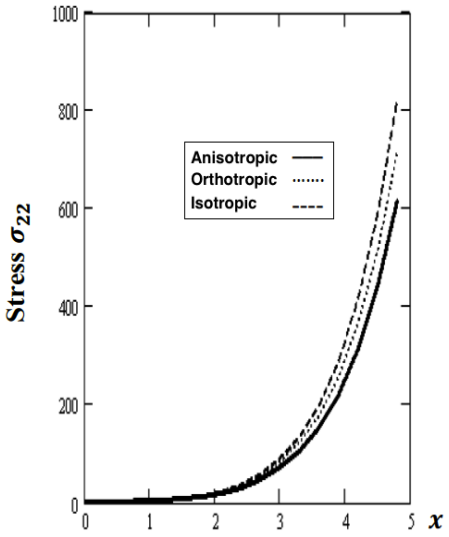

Fig. 7. Variation of the stress σ_{22} with x coordinate for Anisotropic, Orthotropic and Isotropic.

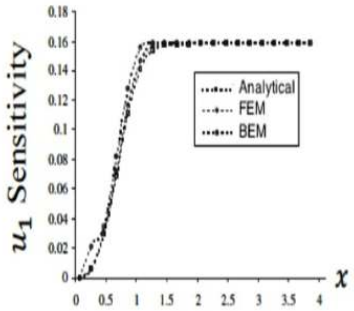

Fig. 8. Variation of the displacement u_1 sensitivity with x coordinate.

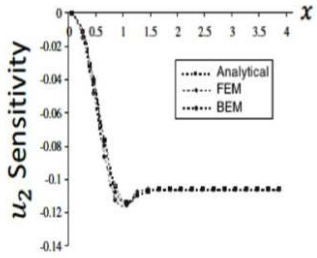

Fig. 9. Variation of the displacement u_2 sensitivity with x coordinate.

7 Conclusion

In the present paper, we demonstrated that the boundary element method can be successfully used in Shape design sensitivity and optimization of anisotropic structures problems. In the proposed method, the effect of

anisotropy on the displacements and stresses is demonstrated and the displacement sensitivities with respect to design variables are calculated using implicit differentiation method (IDM). The numerical optimization method used in the program is the feasible direction approach, together with the golden-section search technique. The shape of anisotropic structures can be manipulated easily by varying a chosen set of the design parameters during the optimization process. The shape sensitivities can be directly derived from the variational form of the governing equations. The accuracy produced by the proposed method enables the use of gradient-based minimizers, that converges superlinearly.

Competing Interests

Author has declared that no competing interests exist.

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