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The (P-A-L) Modified Weibull Distribution

$\begin{array}{ll} \mbox{Mahmoud R. Mahmoud}^{1^*}, \mbox{Rasha M. Mandouh}^1 \\ \mbox{and Marwa M. Hamada}^2 \end{array}$

¹Department of Mathematical Statistics, I.S.S.R., Cairo University, Egypt. ²Department of Statistics, I.S.S.R., Cairo University, Egypt.

Authors' contributions

This work was carried out in collaboration between all authors. All authors read and approved the final manuscript.

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Abstract

This paper presented The (P-A-L) Modified Weibull Distribution. We can compute several properties of this distribution. The maximum likelihood estimators are obtained. Using simulation study, mean, relative bias, root and scaled mean square error for maximum likelihood estimators are obtained. And also, Confidence intervals for unknown two parameters are calculated.

Keywords: Hazard function; maximum likelihood estimation; modified weibull distribution; moment; the (P-A-L) family; variance-covariance matrix.

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^{*}Corresponding author: E-mail: dr_marwa_hamada@yahoo.com;

1 Introduction

Many companies and organizations have too much data and the amount of data available is growing. One of the ways to turn these data into useful information is to have a statistical model for the way the data are generated. On the other hand the industrial processes as well as the way the components of a business interact are getting more and more complicated. For these reasons, no matter how many distributional models become available there arise the needs for new models.

Several methods have been suggested in the statistical literature to construct new statistical distributions from existing ones. Among these methods there is the formula suggested by Marshal and Olkin [1] that became known by their name. There is also the rank transmutation map due to Shaw and Buckley [2]. Of interest to us here is the (P-A-L) extension suggested by Pappas et al. [3]. Lee et al. [4] present a good review of some of the most families methods of generating new families of distributions from existing ones.

In this article, the method known as the (P-A-L) is utilized to create a new family of distributions from the Modified Weibull distribution of Sarahan and Zaindin [5]. This distribution called The (P-A-L) Modified Weibull Distribution with four parameters α, β, γ, p where it denotes as PMWD $(\alpha, \beta, \gamma, p)$. We introduce PMWD $(\alpha, \beta, \gamma, p)$ in section 2 and present properties this distribution in section 3. We estimate four parameters for PMWD by the maximum likelihood estimation. Finally, we analyze real data for PMWD.

2 The (P-A-L) Modified Weibull Distribution

The survival function of the (P-A-L) family is

$$s(x) = \frac{\ln\{1 - (1 - p) s_0(x)\}}{\ln p}, \quad x \in R_+, \ p \in R_+ - \{0\}$$
(2.1)

Where s_0 is the survival function of the base distribution and it should be noted that if $p \to 1$, then $s \to s_0$. The probability density function and hazard function take the form

$$f(x) = \frac{(p-1) f_0(x)}{\{1 - (1-p) s_0(x)\} \ln p}$$
(2.2)

$$h(x) = \frac{(p-1) s_0(x) h_0(x)}{\{1 - (1-p) s_0(x)\} \ln\{1 - (1-p) s_0(x)\}}$$
(2.3)

Where f_0 and h_0 are the probability density function and hazard function of the base distribution. There are several distributions used by the (P-A-L) family, such as the (P-A-L) extended modified Weibull studied by Pappas et al. [3], the (P-A-L) extended Weibull distribution introduced by Al-Zahrani et al. [6] and The (P-A-L) Generalized Exponential Distribution presented by Mahmoud and Mandouh [7].

Now, we put $s_0 = [e^{-\alpha x - \beta x^{\gamma}}]$ is a survival function for the Modified Weibull distribution presented by Sarahan and Zaindin [5].

By substituting s_0 in (2.1), we get

$$S(x) = \frac{\ln(1 - (1 - p)e^{-\alpha x - \beta x^{\gamma}})}{\ln(p)} \quad ; \ x > 0, \ \gamma > 0, \ \alpha, \beta \ge 0, \ p \in R_{+} - \{0\}$$
(2.4)

Hence

$$f(x) = \frac{(p-1)(\alpha + \beta \gamma x^{\gamma - 1})e^{-\alpha x - \beta x^{\gamma}}}{(1 - (1 - p)e^{-\alpha x - \beta x^{\gamma}})\ln(p)}$$
(2.5)

Then the hazard function takes the form

$$h(x) = \frac{(p-1)e^{-\alpha x - \beta x^{\gamma}} [\alpha + \beta \gamma x^{\gamma - 1}]}{[1 - (1-p)e^{-\alpha x - \beta x^{\gamma}}]\ln(1 - (1-p)e^{-\alpha x - \beta x^{\gamma}})}$$
(2.6)

We note that h(x) can be constant, increasing or decreasing depending on the parameter values. For example, if $p \to 1$, $\beta=1$ and $\gamma=1$ then $h(x)=\alpha+1$ is constant, whereas if $p \to 1$ and $\beta=1$, then $h(x) = \alpha + \gamma x^{\gamma-1}$, which is increasing for $\gamma > 1$ and decreasing for $\gamma < 1$.

Figs. 1 and 2 show different shapes for the probability density function and the hazard function of The (P-A-L) Modified Weibull Distribution for different values of the parameters α, β, γ and p.



Fig. 1. Probability density function of the PAL Modified Weibull distribution for different values of the parameters



Fig. 2. Hazard function of the PAL Modified Weibull distribution for different values of the parameters

To find the raw moments, we have

$$E(X^r) = \int_0^\infty x^r f(x) dx$$

from equation(2.5), we get

$$E(X^{r}) = \int_{0}^{\infty} x^{r} \frac{(p-1)(\alpha + \beta \gamma x^{\gamma-1})e^{-\alpha x - \beta x^{\gamma}}}{(1 - (1-p)e^{-\alpha x - \beta x^{\gamma}})\ln(1 - (1-p))} dx$$
(2.7)

To calculate the r^{th} raw moments of The (P-A-L) Modified Weibull Distribution, we can use Numerical integration steps. We can use the Binomial expansion to get the following expression:

$$E(X^{r}) = \sum_{n=0}^{\infty} \frac{(-1)^{n} (p-1)^{(n+1)}}{\log(1-(1-p))} \sum_{i=0}^{\infty} \frac{(-1)^{i}}{i!} \frac{(n+1)^{i} \beta^{i}}{(n+1)^{r+i\gamma}}$$

$$\left[\frac{\Gamma(i\gamma+r+1)}{(n+1)\alpha^{i\gamma+r}} + \frac{\beta\gamma}{(n+1)^{\gamma} \alpha^{\gamma+i\gamma+r}} \Gamma(\gamma+i\gamma+k) \right]$$
(2.8)

Let $\alpha = 1.1, \beta = 2, \gamma = 3, p = 2$ and r = 1, one can easily check that equation (2.8) and numerical integration of (2.7) take the same value, i.e. $E(X^r) = 0.53$.

3 Random Number Generation and Estimation of the Parameters

Using the inversion method, one can generate random from The (P-A-L) Modified Weibull Distribution with the following formula

$$u = F\left(x\right) \tag{3.1}$$

By substituting F(x) in (3.1), we get

$$u = 1 - \frac{\log(1 - (1 - p)e^{-\alpha x - \beta x^{\gamma}})}{\log(p)}$$
(3.2)

Where $u \in (0, 1)$. We take log

$$\beta x^{\gamma} + \alpha x + \log \frac{1 - (1 - (1 - p))^{(1 - u)}}{-(p - 1)} = 0$$
(3.3)

The equation (3.3) has no closed form solution in x_q so, we have to use numerical technique.

Now, we will study parameter estimation by using maximum likelihood.

3.1 Maximum likelihood estimation

Let $x_1, x_2, x_3, ..., x_n$ is a random sample follows The (P-A-L) Modified Weibull Distribution. The likelihood function is given by

$$L(x;\alpha,\beta,\gamma,p) = \prod_{i=1}^{n} \frac{(p-1)(\alpha+\beta\gamma x^{\gamma-1})e^{-\alpha x-\beta x^{\gamma}}}{(1-(1-p)e^{-\alpha x-\beta x^{\gamma}})log(p)}$$
(3.4)

The log likelihood is

$$\ln L = n[ln(p-1)] + \sum_{i=1}^{n} ln(\alpha + \beta \gamma x_i^{\gamma-1}) + \sum_{i=1}^{n} (-\alpha x_i - \beta x_i^{\gamma}) - n[ln(ln(p))] - \sum_{i=1}^{n} ln(1 - (1-p)e^{-\alpha x_i - \beta x_i^{\gamma}})$$
(3.5)

Differentiating (3.5) with respect to α, β, γ and p, we have

$$\frac{\partial \ln L}{\partial \alpha} = \sum_{i=1}^{n} \frac{1}{\alpha + \beta \gamma x_i^{\gamma - 1}} - \sum_{i=1}^{n} x_i + (p - 1) \sum_{i=1}^{n} \frac{x_i e^{-\alpha x_i - \beta x_i^{\gamma}}}{1 - (1 - p) e^{-\alpha x_i - \beta x_i^{\gamma}}}$$
(3.6)

$$\frac{\partial \ln L}{\partial \beta} = \gamma \sum_{i=1}^{n} \frac{x_i^{\gamma - 1}}{\alpha + \beta \gamma x_i^{\gamma - 1}} - \sum_{i=1}^{n} x_i^{\gamma} + (p - 1) \sum_{i=1}^{n} \frac{x_i^{\gamma} e^{-\alpha x_i - \beta x_i^{\gamma}}}{1 - (1 - p) e^{-\alpha x_i - \beta x_i^{\gamma}}}$$
(3.7)

$$\frac{\partial \ln L}{\partial \gamma} = \beta \sum_{i=1}^{n} \frac{x_i^{\gamma - 1} + \gamma(\gamma - 1) x_i^{\gamma - 2}}{\alpha + \beta \gamma x_i^{\gamma - 1}} - \beta \gamma \sum_{i=1}^{n} x_i^{\gamma - 1} + (p - 1) \beta \gamma \sum_{i=1}^{n} \frac{x_i^{\gamma - 1} e^{-\alpha x_i - \beta x_i^{\gamma}}}{1 + (p - 1) e^{-\alpha x_i - \beta x_i^{\gamma}}} \quad (3.8)$$

$$\frac{\partial \ln L}{\partial p} = \frac{n}{(p-1)} - \frac{n}{\{1 - (1-p)\}\{\ln(1 - (1-p))\}} - \sum_{i=1}^{n} \frac{e^{-\alpha x_i - \beta x_i^{\gamma}}}{1 - (1-p)e^{-\alpha x_i - \beta x_i^{\gamma}}}$$
(3.9)

Equating the derivatives in (3.6), (3.7), (3.8) and (3.9) to zero, then we solve the four nonlinear equations by numerically, we get the maximum likelihood estimators $\hat{\alpha}, \hat{\beta}, \hat{\gamma}$ and \hat{p} .

Hence

The second derivatives of the log likelihood function are given by

$$\begin{split} \frac{\partial^2 \ln L}{\partial \alpha^2} &= -I_{11} = -\Sigma_{i=1}^n (-\frac{e^{-2\alpha x_i - 2\beta x_i^{\gamma}} (p-1)^2 x_i^2}{(1+e^{-\alpha x_i - \beta x_i^{\gamma}} (p-1))^2} + \frac{e^{-\alpha x_i - \beta x_i^{\gamma}} (p-1) x_i^2}{1+e^{-\alpha x_i - \beta x_i^{\gamma}} (p-1)}) \\ &+ \Sigma_{i=1}^n - \frac{1}{(\alpha + \beta \gamma x_i^{\gamma - 1})^2} \\ \frac{\partial^2 \ln L}{\partial \beta^2} &= -I_{22} = -\Sigma_{i=1}^n \frac{\gamma^2 x_i^{-2 - 2\gamma}}{(\alpha + \beta \gamma x_i^{-1 + \gamma})^2} - -\Sigma_{i=1}^n (-\frac{e^{-2\alpha x_i - 2\beta x_i^{\gamma}} (p-1)^2 x_i^{2\gamma}}{(1+e^{-\alpha x_i - \beta x_i^{\gamma}} (p-1))^2} \\ &+ \frac{e^{-\alpha x_i - \beta x_i^{\gamma}} (p-1) x_i^{2\gamma}}{1+e^{-\alpha x_i - \beta x_i^{\gamma}} (p-1)}) \\ \frac{\partial^2 \ln L}{\partial \gamma^2} &= -I_{33} = \Sigma_{i=1}^n - \beta \log[x_i]^2 x_i^{\gamma} + \Sigma_{i=1}^n [-\frac{(\beta x_i^{-1 + \gamma} + \beta \gamma \log[x_i] x_i^{-1 + \gamma})^2}{(\alpha + \beta \gamma x_i^{-1 + \gamma})^2} \end{split}$$

$$+\frac{2\beta\log[x_{i}]x_{i}^{-1+\gamma}+\beta\gamma\log[x_{i}]^{2}x_{i}^{-1+\gamma}}{\alpha+\beta\gamma x_{i}^{-1+\gamma}}]-\Sigma_{i=1}^{n}[-\frac{e^{-\alpha x_{i}-\beta x_{i}^{\gamma}}\beta(p-1)\log[x_{i}]^{2}x_{i}^{\gamma}}{1+e^{-\alpha x_{i}-\beta x_{i}^{\gamma}}(p-1)}$$

$$\begin{split} &-\frac{e^{-2\alpha x_{i}-2\beta x_{i}^{\gamma}}\beta^{2}(p-1)^{2}\log[x_{i}]^{2}x_{i}^{2}}{(1+e^{-\alpha x_{i}-\beta x_{i}^{\gamma}}(p-1))(p-1)^{2}}+\frac{e^{-\alpha x_{i}-\beta x_{i}^{\gamma}}(p-1)\log[x_{i}]^{2}x_{i}^{2}\gamma}{1+e^{-\alpha x_{i}-\beta x_{i}^{\gamma}}(p-1)}\right]\\ &\frac{\partial^{2}\ln L}{\partial p^{2}}=-I_{44}=-\frac{n}{(p-1)^{2}}+\frac{n}{(1-(1-p))^{2}}\log[1-(1-p)]^{2}+\frac{n}{(1-(1-p))^{2}}\log[1-(1-p)]}{-\sum_{i=1}^{n}(\frac{-e^{-2\alpha x_{i}-2\beta x_{i}^{\gamma}}}{(1+e^{-\alpha x_{i}-\beta x_{i}^{\gamma}}(p-1))^{2}}+\frac{e^{\alpha x_{i}-\beta x_{i}^{\gamma}}(p-1)x_{i}^{1+\gamma}}{(1+e^{-\alpha x_{i}-\beta x_{i}^{\gamma}}(p-1))^{2}}\\ &\frac{\partial^{2}lnL}{\partial \alpha \partial \beta}=-I_{12}=\sum_{i=1}^{n}\frac{-\gamma x_{i}^{-1+\gamma}}{(\alpha+\beta\gamma x_{i}^{-1+\gamma})^{2}}-\sum_{i=1}^{n}(-\frac{e^{-2\alpha x_{i}-2\beta x_{i}^{\gamma}}(p-1)^{2}x_{i}^{1+\gamma}}{(1+e^{-\alpha x_{i}-\beta x_{i}^{\gamma}}(p-1))^{2}}+\frac{e^{\alpha x_{i}-\beta x_{i}^{\gamma}}(p-1)x_{i}^{1+\gamma}}{1+e^{-\alpha x_{i}-\beta x_{i}^{\gamma}}(p-1))^{2}}\\ &\frac{\partial^{2}lnL}{\partial \alpha \partial \gamma}=-I_{13}=\sum_{i=1}^{n}-\frac{\beta x_{i}^{-1+\gamma}+\beta \gamma \log[x_{i}]x_{i}^{-1+\gamma}}{(1+e^{-\alpha x_{i}-\beta x_{i}^{\gamma}}(p-1))}\sum_{i=1}^{n}\left[-\frac{e^{-2\alpha x_{i}-2\beta x_{i}^{\gamma}}\beta(p-1)^{2}\log[x_{i}]x_{i}^{1+\gamma}}{(1+e^{-\alpha x_{i}-\beta x_{i}^{\gamma}}(p-1))^{2}}\right]\\ &+\frac{e^{-\alpha x_{i}-\beta x_{i}^{\gamma}}\beta(p-1)\log[x_{i}]x_{i}^{1+\gamma}}{1+e^{-\alpha x_{i}-\beta x_{i}^{\gamma}}(p-1)}\right]\\ &\frac{\partial^{2}lnL}{\partial \alpha \partial p}=-I_{14}=-\sum_{i=1}^{n}\left(\frac{e^{-2\alpha x_{i}-2\beta x_{i}^{\gamma}}(p-1)\log[x_{i}]x_{i}^{2}\gamma}{1+e^{-\alpha x_{i}-\beta x_{i}^{\gamma}}(p-1)}\right)\\ &\frac{\partial^{2}lnL}{\partial \beta \partial p}=-I_{23}=\sum_{i=1}^{n}-\log[x_{i}]x_{i}^{\gamma}-\sum_{i=1}^{n}\left[-\frac{e^{-\alpha x_{i}-\beta x_{i}^{\gamma}}(p-1)\log[x_{i}]x_{i}^{\gamma}}{1+e^{-\alpha x_{i}-\beta x_{i}^{\gamma}}(p-1)}\right]\\ &-\frac{e^{-2\alpha x_{i}-2\beta x_{i}^{\gamma}}\beta(p-1)^{2}\log[x_{i}]x_{i}^{2}\gamma}}{(1+e^{-\alpha x_{i}-\beta x_{i}^{\gamma}}(p-1))^{2}}+\frac{e^{-\alpha x_{i}-\beta x_{i}^{\gamma}}\beta(p-1)\log[x_{i}]x_{i}^{\gamma}}{1+e^{-\alpha x_{i}-\beta x_{i}^{\gamma}}(p-1)}\right]\\ &\frac{\partial^{2}lnL}{\partial \beta \partial p}=-I_{24}=-\sum_{i=1}^{n}\left(\frac{e^{-2\alpha x_{i}-2\beta x_{i}^{\gamma}}\beta(p-1)x_{i}^{\gamma}}{(1+e^{-\alpha x_{i}-\beta x_{i}^{\gamma}}(p-1))^{2}}-\frac{e^{-\alpha x_{i}-\beta x_{i}^{\gamma}}x_{i}^{\gamma}}{1+e^{-\alpha x_{i}-\beta x_{i}^{\gamma}}(p-1)}\right)\\ &\frac{\partial^{2}lnL}{\partial \beta \partial p}=-I_{34}=-\sum_{i=1}^{n}\left(\frac{e^{-2\alpha x_{i}-2\beta x_{i}^{\gamma}}\beta(p-1)x_{i}^{\gamma}}{(1+e^{-\alpha x_{i}-\beta x_{i}^{\gamma}}(p-1))^{2}}-\frac{e^{-\alpha x_{i}-\beta x_{i}^{\gamma}}x_{i}^{\gamma}}{1+e^{-\alpha x_{i}-\beta x_{i}^{\gamma}}x_{i}^{\gamma}}\right)\\ &\frac{\partial^{2}lnL}{\partial \beta \partial p}=-I_{34}=-\sum_{i=1}^{n}\left(\frac{e^{-2\alpha x_{i}-2\beta x_{i}^{\gamma}}\beta(p-1)x_{i$$

Then the observed information matrix is given by

$$I = \begin{pmatrix} I_{11} & I_{12} & I_{13} & I_{14} \\ I_{21} & I_{22} & I_{23} & I_{24} \\ I_{31} & I_{32} & I_{33} & I_{34} \\ I_{41} & I_{42} & I_{43} & I_{44} \end{pmatrix}$$

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So that the variance-covariance matrix may be approximated as

$$V = \begin{pmatrix} V_{11} & V_{12} & V_{13} & V_{14} \\ V_{21} & V_{22} & V_{23} & V_{24} \\ V_{31} & V_{32} & V_{33} & V_{34} \\ V_{41} & V_{42} & V_{43} & V_{44} \end{pmatrix} = \begin{pmatrix} I_{11} & I_{12} & I_{13} & I_{14} \\ I_{21} & I_{22} & I_{23} & I_{24} \\ I_{31} & I_{32} & I_{33} & I_{34} \\ I_{41} & I_{42} & I_{43} & I_{44} \end{pmatrix}^{-1}$$

It is known that the asymptotic distribution of the MLE $(\hat{\alpha}, \hat{\beta}, \hat{\gamma}, \hat{p})$ is given by

$$\begin{pmatrix} \hat{\alpha} \\ \hat{\beta} \\ \hat{\gamma} \\ \hat{p} \end{pmatrix} \sim N \begin{bmatrix} \begin{pmatrix} \alpha \\ \beta \\ \gamma \\ p \end{pmatrix}, \begin{pmatrix} V_{11} & V_{12} & V_{13} & V_{14} \\ V_{21} & V_{22} & V_{23} & V_{24} \\ V_{31} & V_{32} & V_{33} & V_{34} \\ V_{41} & V_{42} & V_{43} & V_{44} \end{pmatrix}$$
 (3.10)

Since V involves the parameters α, β, γ, p , we replace the parameters by the corresponding MLE's in order to obtain an estimate of V, which is denoted by

$$\hat{V} = \begin{pmatrix}
I_{11} & I_{12} & I_{13} & I_{14} \\
I_{21} & I_{22} & I_{23} & I_{24} \\
I_{31} & I_{32} & I_{33} & I_{34} \\
I_{41} & I_{42} & I_{43} & I_{44}
\end{pmatrix}^{-1}$$
(3.11)

Where \hat{I}_{ij} when $(\hat{\alpha}, \hat{\beta}, \hat{\gamma}, \hat{p})$ replaces $(\alpha, \beta, \gamma, p)$.

By using (3.10), approximate $100(1-\theta)\%$ confidence intervals for α, β, γ, p are determined, respectively, as

$$\hat{\alpha} \pm z_{\theta/2} \sqrt{\hat{V}_{11}}, \quad \hat{\beta} \pm z_{\theta/2} \sqrt{\hat{V}_{22}}, \quad \hat{\gamma} \pm z_{\theta/2} \sqrt{\hat{V}_{33}}, \quad \hat{p} \pm z_{\theta/2} \sqrt{\hat{V}_{44}}$$
 (3.12)

Where z_{θ} is the upper θ -th percentile of the standard normal distribution.

4 Simulation Study

We used a simulation study to check the performance of the accuracy of point and interval estimates for Several cases, of which estimates two parameters, estimates three parameters and finally, estimates four parameters of $PMWD(\alpha, \beta, \gamma, p)$ for m=1000, the sample size n are 50, 100, 150, 200, 250, 300 and different parameter values. The following steps were followed to obtain the results:

- 1. Specify initial values for parameters α, β, γ and p.
- 2. Specify the sample size n.
- 3. Generate m times of random sample with size n from $PMWD(\alpha, \beta, \gamma, p)$.
- 4. Obtain the maximum likelihood estimates for α, β, γ, p for different sample sizes.

5. Obtain the mean, bias and relative bias, mean squared error (MSE), root mean squared error and scaled root mean square error for each estimator for different sample size.

6. Repeat 1:5 for several values of α, β, γ and p.

By the software Mathematica 10 we tried to do simulation and estimate the parameters (γ, p) with respect to (α, β) are known, The result is shown in Table 1 and we estimate the parameters (β, p) with respect to (α, γ) are known as shown in Table 2. Similarly, we estimate the parameters (β, γ, p) with respect to α are known as shown in Table 3. Finally, we estimate the parameters $(\alpha, \beta, \gamma, p)$ as shown in Table 4.

Based on the values we obtained from Simulation study, the results are shown in Tables 1, 2, 3 and 4.

	1. $\alpha_0 = 1.25, \beta_0 = 2.5, \gamma_0 = 1.5, p_0 = 1.98$								
		Mean	Relative bias	variance	RMSE	Scaled RMSE	$\mathbf{L}\mathbf{L}$	UL	
n=50	γ	1.64	-0.09	0.25	0.52	0.35	1.03	2.25	
	p	2.41	-0.22	3.52	1.92	0.97	-0.83	5.64	
n=75	γ	1.60	-0.7	0.22	0.48	0.32	0.88	2.70	
	p	2.38	-0.20	2.52	1.64	0.83	-2.47	7.24	
n = 100	γ	1.59	-0.06	0.18	0.44	0.27	1.08	0.39	
	p	2.30	-0.16	1.96	1.44	0.62	2.10	4.20	
n = 150	γ	1.57	-0.05	0.12	0.35	0.23	1.18	1.96	
	p	2.19	-0.11	1.26	1.14	0.52	0.81	3.58	
			2. $\alpha_0 = 1.25$,	$\beta_0=2.5,\gamma$	$r_0=0.5, p$	$p_0 = 0.9$			
		Mean	Relative bias	variance	RMSE	Scaled RMSE	$\mathbf{L}\mathbf{L}$	UL	
n=50	γ	0.54	-0.04	0.007	0.09	0.18	0.06	1.02	
	\mathbf{p}	0.98	-0.08	0.490	0.71	0.72	-3.47	5.45	
n=75	γ	0.53	-0.06	0.005	0.08	0.15	0.41	0.65	
	\mathbf{p}	0.97	-0.08	0.350	0.60	0.62	-0.56	2.52	
n = 100	γ	0.52	-0.05	0.003	0.06	0.12	0.46	0.59	
	\mathbf{p}	0.94	-0.04	0.180	0.43	0.46	-0.05	1.92	
n = 150	γ	0.51	-0.03	0.002	0.05	0.09	0.29	0.75	
	\mathbf{p}	0.93	-0.04	0.130	0.36	0.39	-1.36	3.24	
			3. $\alpha_0 = 1.2$	$25, \beta_0 = 2.5$	$\gamma_0=7,\gamma_0$	$p_0 = 5$			
		Mean	Relative bias	variance	RMSE	Scaled RMSE	$\mathbf{L}\mathbf{L}$	UL	
n=50	γ	7.18	-0.03	02.87	1.71	0.23	4.24	10.13	
	\mathbf{p}	5.85	-0.17	10.25	3.31	0.56	0.41	11.31	
n=75	γ	7.34	-0.04	02.45	1.60	0.22	4.76	09.93	
	\mathbf{p}	5.47	-0.09	06.85	2.66	0.48	0.89	10.05	
n = 100	γ	7.19	-0.02	01.69	1.32	0.18	2.63	11.76	
	\mathbf{p}	5.46	-0.09	05.87	2.46	0.45	-6.57	17.49	
n = 150	γ	7.09	-0.01	01.04	1.02	0.14	4.76	09.42	
	\mathbf{p}	5.26	-0.05	03.46	1.88	0.36	1.68	08.86	

Table 1. MLE's and confidence intervals of two parameter (γ and p) in the case of the (P-A-L) modified Weibull distribution

Table 2. MLE's and confidence intervals of two parameter (β and p) in the case of the (P-A-L) modified Weibull distribution

			1. $\alpha_0 = 1.25$,	$\beta_0 = 2.5, \gamma_0$	$p = 1.5, p_0$	$_{0} = 1.98$		
		Mean	Relative bias	variance	RMSE	Scaled RMSE	\mathbf{LL}	UL
n=50	β	2.93	-0.17	00.89	1.04	0.42	0.92	4.95
	р	2.49	-0.26	15.35	3.95	1.99	-3.88	8.88
n=75	β	2.79	-0.12	00.49	0.76	0.30	0.92	4.66
	\mathbf{p}	1.93	0.02	02.41	1.55	0.78	-2.25	6.12
n = 100	β	2.74	-0.09	00.35	0.64	0.14	1.59	3.88
	\mathbf{p}	1.78	0.09	01.43	1.21	0.61	0.14	3.45
n = 150	β	2.67	-0.07	00.24	0.52	0.21	1.17	4.17
	\mathbf{p}	1.59	0.19	00.47	0.78	0.39	-0.75	3.94
			2. $\alpha_0 = 1.2$	$(5, \beta_0 = 0.5)$	$, \gamma_0 = 1.5$	$, p_0 = 2$		
		Mean	Relative bias	variance	RMSE	Scaled RMSE	LL	UL
n=50	β	0.69	-0.39	00.08	0.35	0.72	-00.67	02.07
	\mathbf{p}	3.43	-0.72	22.32	4.93	2.46	-11.83	18.83
n=75	β	0.65	-0.30	00.06	0.28	0.57	-00.23	01.53
	\mathbf{p}	2.92	-0.46	09.53	3.22	1.61	-03.71	09.55
n=100	β	0.61	-0.21	00.03	0.21	0.41	00.15	01.05
	\mathbf{p}	2.49	-0.24	01.95	1.47	0.74	-00.01	04.98
n = 150	β	0.58	-0.16	00.02	0.17	0.35	00.23	00.94
	\mathbf{p}	2.34	-0.17	00.88	0.99	0.49	00.54	04.13
			3. $\alpha_0 = 1.25$,	$\beta_0 = 1.5, \gamma$	$p_0 = 1.5, p$	$p_0 = 0.5$		
		Mean	Relative bias	variance	RMSE	Scaled RMSE	LL	UL
n=50	β	1.69	-0.13	0.46	0.71	0.47	0.11	3.27
	\mathbf{p}	0.84	-0.68	1.05	1.07	2.16	-0.74	2.43
n = 75	β	1.64	-0.09	0.25	0.52	0.35	0.68	2.59
	\mathbf{p}	0.72	-0.44	0.21	0.51	1.02	0.08	1.35
n = 100	β	1.59	-0.06	0.17	0.42	0.28	0.07	3.11
	\mathbf{p}	0.65	-0.29	0.09	0.33	0.67	-0.67	1.97
n = 150	β	1.55	-0.04	0.11	0.33	0.22	0.75	2.36
	\mathbf{p}	0.61	-0.21	0.04	0.22	0.43	0.05	1.16

		1. α_0	$= 1.25, \beta_0 = 2.$	$5, \gamma_0 = 0.5$	$p_0 = 0.9$	
<u> </u>		Mean	Belative bias	variance	BMSE	Scaled BMSE
50	B	2 738	0.095	0.568	0.791	0.316
n=30		2.738	-0.093	0.005	0.791	0.310
	- Y	1 285	-0.428	0.784	0.071	1.073
n = 100	B	2.670	-0.068	0.312	0.584	0.234
11-100	~	0.496	0.007	0.003	0.054	0.107
	Ď	1.216	-0.351	0.457	0.746	0.829
n = 150	B	2.627	-0.051	0.167	0.427	0.171
	γ	0.496	0.006	0.002	0.048	0.097
	\dot{p}	1.142	-0.268	0.320	0.615	0.684
n=200	B	2.627	-0.051	0.167	0.427	0.171
	γ	0.499	0.002	0.002	0.043	0.087
	p	1.120	-0.245	0.306	0.596	0.662
n=250	B	2.587	-0.035	0.149	0.395	0.158
	γ	0.498	0.004	0.001	0.038	0.076
	p	1.088	-0.209	0.268	0.551	0.612
n=300	B	2.578	-0.031	0.133	0.373	0.149
	γ	0.498	0.004	0.001	0.035	0.071
	p	1.076	-0.196	0.245	0.525	0.584
		$2. \alpha_0$	$\beta_0 = 0.2, \beta_0 = 2.2$	$2, \gamma_0 = 2.8,$	$p_0 = 0.8$	
		Mean	Relative bias	variance	\mathbf{RMSE}	Scaled RMSE
n=50	β	2.529	-0.149	0.305	0.625	0.292
	γ	2.719	0.029	0.211	0.466	0.167
	\mathbf{p}	1.461	-0.0826	1.042	1.216	1.520
n=100	B	2.428	-0.104	0.149	0.449	0.204
	γ	2.720	0.028	0.151	0.397	0.142
	\mathbf{p}	1.294	-0.618	0.707	0.975	1.219
n = 150	B	2.388	-0.085	0.104	0.374	0.169
	γ	2.722	0.028	0.109	0.339	0.121
	\mathbf{p}	1.230	-0.538	0.581	0.875	1.094
n=200	β	2.359	-0.072	0.086	0.333	0.151
	γ	2.701	0.035	0.082	0.303	0.108
	$\frac{p}{2}$	1.176	-0.470	0.362	0.709	0.887
n=250	13	2.342	-0.065	0.076	0.311	0.141
	Y	2.739	0.022	0.081	0.292	0.104
n-300	B	2 3/3	-0.065	0.065	0.711	0.333
m=300	~	2.719	0.029	0.071	0.278	0.099
	p	1.127	-0.409	0.320	0.654	0.817
-	*					
			1.0.0	2 5	0.5	
		3.	$\alpha_0 = 1.2, \beta_0 = 1$	$2, \gamma_0 = 5, p_0$	0 = 6.5	<u>a 1 1 5 1 6 5</u>
		3. Mean	$ \alpha_0 = 1.2, \beta_0 = $ Relative bias	$\frac{2, \gamma_0 = 5, p_0}{\text{variance}}$	0 = 6.5 RMSE	Scaled RMSE
n=50	β	3. Mean 02.353	$ \alpha_0 = 1.2, \beta_0 = $ Relative bias -0.176	$2, \gamma_0 = 5, p_0$ variance 000.428	0 = 6.5 RMSE 00.744	Scaled RMSE
n=50	β γ	3. Mean 02.353 05.134	$\frac{\alpha_0 = 1.2, \beta_0 =}{\text{Relative bias}}$ $\begin{array}{c} -0.176\\ -0.026\\ -0.026\end{array}$	$2, \gamma_0 = 5, p_0$ variance 000.428 003.262	0 = 6.5 RMSE 00.744 01.811	Scaled RMSE 0.372 0.362
n=50	β γ P	3. Mean 02.353 05.134 12.865	$\begin{array}{l} \alpha_0 = 1.2, \beta_0 = \\ \hline \text{Relative bias} \\ -0.176 \\ -0.026 \\ -0979 \\ 0.110 \end{array}$	$\frac{2, \gamma_0 = 5, p_0}{\text{variance}} \\ \frac{000.428}{003.262} \\ 141.060 \\ 200.927 \\ 141.060 \\ 200.927 \\ 141.060 \\ 200.927 \\ 141.060 \\ 200.927 \\ 141.060 \\ 200.927 \\ 141.060 \\ 200.927 \\ 141.060 \\ 200.927 \\ 141.060 \\ 200.927 \\ 141.060 \\ 200.927 \\ 141.060 \\ 200.927 \\ 141.060 \\ 200.927 \\ 141.060 \\ 200.927 \\ 141.060 \\ 200.927 \\ 141.060 \\ 200.927 \\ 141.060 \\ 200.927 \\ 141.060 \\ 200.927 \\ 141.060 \\ 200.927 \\ 141.060 \\ 200.927 \\ 141.060 \\ 200.927 $	b = 6.5 RMSE 00.744 01.811 13.475 05.500	Scaled RMSE 0.372 0.362 2.073
n=50 n=100	$egin{array}{c} eta \ \gamma \ \mathbf{p} \ eta \ eba \ eba \ eba \ $	3. Mean 02.353 05.134 12.865 02.237 04.034	$\begin{array}{c} \alpha_0 = 1.2, \beta_0 = \\ \hline \text{Relative bias} \\ -0.176 \\ -0.026 \\ -0979 \\ -0.118 \\ 0.022 \end{array}$	$\frac{2, \gamma_0 = 5, p_0}{\text{variance}} \\ \frac{000.428}{003.262} \\ 141.060 \\ 000.267 \\ 000.417 $	b = 6.5 RMSE 00.744 01.811 13.475 00.569 01 199	Scaled RMSE 0.372 0.362 2.073 0.284 0.284
n=50 n=100	$\beta \\ \gamma \\ \mathbf{p} \\ \beta \\ \gamma$	3. Mean 02.353 05.134 12.865 02.237 04.984 10 502	$\begin{array}{c} \alpha_0 = 1.2, \beta_0 = \\ \hline \text{Relative bias} \\ -0.176 \\ -0.026 \\ -0979 \\ -0.118 \\ 0.033 \\ 0.616 \end{array}$	$\begin{array}{c} 2, \gamma_0 = 5, p_0 \\ \hline \text{variance} \\ 000.428 \\ 003.262 \\ 141.060 \\ 000.267 \\ 001.414 \\ 071.422 \end{array}$	$\begin{array}{c} \mathbf{p} = 6.5 \\ \hline \mathbf{RMSE} \\ 00.744 \\ 01.811 \\ 13.475 \\ 00.569 \\ 01.189 \\ 00.259 \end{array}$	Scaled RMSE 0.372 0.362 2.073 0.284 0.238
n=50 n=100	$\beta \\ \gamma \\ \mathbf{p} \\ \beta \\ \gamma \\ \mathbf{p} \\ \beta \\ \gamma \\ \mathbf{p} \\ \boldsymbol{\rho} \\ \boldsymbol{\rho}$	3. Mean 02.353 05.134 12.865 02.237 04.984 10.503 02.159	$\begin{array}{c} \alpha_0 = 1.2, \beta_0 = \\ \hline \text{Relative bias} \\ -0.176 \\ -0.026 \\ -0979 \\ -0.118 \\ 0.033 \\ -0.616 \\ 0.070 \end{array}$	$\begin{array}{c} 2, \gamma_0 = 5, p_0 \\ \hline \text{variance} \\ 000.428 \\ 003.262 \\ 141.060 \\ 000.267 \\ 001.414 \\ 071.433 \\ 000.105 \end{array}$	$\begin{array}{c} \mathbf{p} = 6.5 \\ \hline \mathbf{RMSE} \\ 00.744 \\ 01.811 \\ 13.475 \\ 00.569 \\ 01.189 \\ 09.352 \\ 09.352 \end{array}$	Scaled RMSE 0.372 0.362 2.073 0.284 0.238 1.439 0.225
n=50 n=100 n=150	β γ P β γ P β γ	3. Mean 02.353 05.134 12.865 02.237 04.984 10.503 02.158 04.953	$\begin{array}{l} \alpha_0 = 1.2, \beta_0 = \\ \hline \text{Relative bias} \\ -0.176 \\ -0.026 \\ -0.979 \\ -0.118 \\ 0.033 \\ -0.616 \\ -0.079 \\ 0.000 \end{array}$	$\begin{array}{c} 2, \gamma_0 = 5, p_0 \\ \hline \text{variance} \\ 000.428 \\ 003.262 \\ 141.060 \\ 000.267 \\ 001.414 \\ 071.433 \\ 000.195 \\ 000.903 \end{array}$	$\begin{array}{c} \mathbf{p} = 6.5 \\ \hline \mathbf{RMSE} \\ 00.744 \\ 01.811 \\ 13.475 \\ 00.569 \\ 01.189 \\ 09.352 \\ 00.469 \\ 00.952 \end{array}$	Scaled RMSE 0.372 0.362 2.073 0.284 0.238 1.439 0.235 0.190
n=50 n=100 n=150	B Y P B Y P B Y P B Y P B Y	3. Mean 02.353 05.134 12.865 02.237 04.984 10.503 02.158 04.953 09.652	$\begin{array}{l} \alpha_0 = 1.2, \beta_0 = \\ \hline \text{Relative bias} \\ -0.176 \\ -0.026 \\ -0979 \\ -0.118 \\ 0.033 \\ -0.616 \\ -0.079 \\ 0.009 \\ -0.485 \end{array}$	$\begin{array}{c} 2, \gamma_0 = 5, p_0 \\ \hline \text{variance} \\ 000.428 \\ 003.262 \\ 141.060 \\ 000.267 \\ 001.414 \\ 071.433 \\ 000.195 \\ 000.903 \\ 053.834 \end{array}$	$\begin{array}{l} \mathbf{p} = 6.5 \\ \mathbf{RMSE} \\ 00.744 \\ 01.811 \\ 13.475 \\ 00.569 \\ 01.189 \\ 09.352 \\ 00.469 \\ 00.952 \\ 07.985 \end{array}$	Scaled RMSE 0.372 0.362 2.073 0.284 0.238 1.439 0.235 0.190 1.229
n=50 n=100 n=150 n=200	$ \begin{array}{c} \beta \\ \gamma \\ p \\ \beta \\ \gamma \\ p \\ \beta \\ \gamma \\ p \\ \beta \\ \beta \end{array} $	$\begin{array}{r} 3.\\ \text{Mean}\\ 02.353\\ 05.134\\ 12.865\\ 02.237\\ 04.984\\ 10.503\\ 02.158\\ 04.953\\ 09.652\\ 02.141\\ \end{array}$	$\begin{array}{l} \alpha_0 = 1.2, \beta_0 = \\ \hline \text{Relative bias} \\ -0.176 \\ -0.026 \\ -0979 \\ -0.118 \\ 0.033 \\ -0.616 \\ -0.079 \\ 0.009 \\ -0.485 \\ -0.070 \end{array}$	$\begin{array}{c} 2, \gamma_0 = 5, p_0 \\ \hline \text{variance} \\ 000.428 \\ 003.262 \\ 141.060 \\ 000.267 \\ 001.414 \\ 071.433 \\ 000.195 \\ 000.903 \\ 053.834 \\ 000 167 \end{array}$	$\begin{array}{c} \mathbf{p} = 6.5 \\ \mathbf{RMSE} \\ 00.744 \\ 01.811 \\ 13.475 \\ 00.569 \\ 01.189 \\ 09.352 \\ 00.469 \\ 00.952 \\ 07.985 \\ 00.433 \end{array}$	Scaled RMSE 0.372 0.362 2.073 0.284 0.238 1.439 0.235 0.190 1.229 0.216
n=50 n=100 n=150 n=200	$ \begin{array}{c} \beta \\ \gamma \\ p \\ \beta \\ \gamma \\ \rho \\ \beta \\ \gamma \\ \rho \\ \beta \\ \gamma \\ \rho \\ \rho$	$\begin{array}{r} 3.\\ \text{Mean}\\ 02.353\\ 05.134\\ 12.865\\ 02.237\\ 04.984\\ 10.503\\ 02.158\\ 04.953\\ 09.652\\ 02.141\\ 04.987\end{array}$	$\begin{array}{r} \alpha_0 = 1.2, \beta_0 = \\ \hline \text{Relative bias} \\ -0.176 \\ -0.026 \\ -0979 \\ -0.118 \\ 0.033 \\ -0.616 \\ -0.079 \\ 0.009 \\ -0.485 \\ -0.070 \\ 0.003 \end{array}$	$\begin{array}{c} 2, \gamma_0 = 5, p_0 \\ \hline \text{variance} \\ 000.428 \\ 003.262 \\ 141.060 \\ 000.267 \\ 001.414 \\ 071.433 \\ 000.195 \\ 000.903 \\ 053.834 \\ 000.167 \\ 000.691 \end{array}$	$\begin{array}{c} \mathbf{b} = 6.5 \\ \hline \mathbf{RMSE} \\ 00.744 \\ 01.811 \\ 13.475 \\ 00.569 \\ 01.189 \\ 09.352 \\ 00.469 \\ 00.952 \\ 07.985 \\ 00.433 \\ 00.832 \end{array}$	$\begin{array}{c} {\rm Scaled\ RMSE} \\ 0.372 \\ 0.362 \\ 2.073 \\ 0.284 \\ 0.238 \\ 1.439 \\ 0.235 \\ 0.190 \\ 1.229 \\ 0.216 \\ 0.216 \\ 0.166 \end{array}$
n=50 n=100 n=150 n=200	$ \begin{array}{c} \beta \\ \gamma \\ \mathbf{p} \\ \beta \\ \gamma \\ \mathbf{p} \\ \beta \\ \gamma \\ p \\ p \\ \beta \\ \gamma \\ p \\ \gamma \\ \gamma$	$\begin{array}{c} 3.\\ \text{Mean}\\ 02.353\\ 05.134\\ 12.865\\ 02.237\\ 04.984\\ 10.503\\ 02.158\\ 04.953\\ 09.652\\ 02.141\\ 04.987\\ 09.101\\ \end{array}$	$\begin{array}{l} \alpha_0 = 1.2, \beta_0 = \\ \hline \text{Relative bias} \\ -0.176 \\ -0.026 \\ -0979 \\ -0.118 \\ 0.033 \\ -0.616 \\ -0.079 \\ 0.009 \\ -0.485 \\ -0.070 \\ 0.003 \\ -0.003 \\ -0.400 \end{array}$	$\begin{array}{c} 2, \gamma_0 = 5, p_0 \\ \hline \text{variance} \\ 000.428 \\ 003.262 \\ 141.060 \\ 000.267 \\ 001.414 \\ 071.433 \\ 000.195 \\ 000.903 \\ 053.834 \\ 000.167 \\ 000.691 \\ 000.691 \\ 049.919 \end{array}$	$\begin{array}{l} \mathbf{p} = 6.5 \\ \mathbf{RMSE} \\ 00.744 \\ 01.811 \\ 13.475 \\ 00.569 \\ 01.189 \\ 09.352 \\ 00.469 \\ 00.952 \\ 07.985 \\ 00.433 \\ 00.832 \\ 07.529 \end{array}$	$\begin{array}{c} \text{Scaled RMSE} \\ 0.372 \\ 0.362 \\ 2.073 \\ 0.284 \\ 0.238 \\ 1.439 \\ 0.235 \\ 0.190 \\ 1.229 \\ 0.216 \\ 0.166 \\ 1.158 \end{array}$
n=50 n=100 n=150 n=200 n=250	$ \begin{array}{c} \beta \\ \gamma \\ \mathbf{p} \\ \beta \\ \gamma \\ \mathbf{p} \\ \beta \\ \gamma \\ p \\ \beta \\ \gamma \\ p \\ \beta \\ \beta \end{array} $	$\begin{array}{c} 3.\\ \text{Mean}\\ 02.353\\ 05.134\\ 12.865\\ 02.237\\ 04.984\\ 10.503\\ 02.158\\ 04.953\\ 09.652\\ 02.141\\ 04.987\\ 09.101\\ 02.084 \end{array}$	$\begin{array}{r} \alpha_0 = 1.2, \beta_0 = \\ \hline \text{Relative bias} \\ -0.176 \\ -0.026 \\ -0.979 \\ -0.118 \\ 0.033 \\ -0.616 \\ -0.079 \\ 0.009 \\ -0.485 \\ -0.070 \\ 0.003 \\ -0.400 \\ -0.400 \\ -0.042 \end{array}$	$\begin{array}{c} 2, \gamma_0 = 5, p_0 \\ \hline \text{variance} \\ 000.428 \\ 003.262 \\ 141.060 \\ 000.267 \\ 001.414 \\ 071.433 \\ 000.195 \\ 000.903 \\ 053.834 \\ 000.167 \\ 000.691 \\ 049.919 \\ 000.126 \end{array}$	$\begin{array}{c} \mathbf{p} = 6.5 \\ \mathbf{RMSE} \\ 00.744 \\ 01.811 \\ 13.475 \\ 00.569 \\ 01.189 \\ 09.352 \\ 00.469 \\ 00.952 \\ 07.985 \\ 00.433 \\ 00.832 \\ 07.529 \\ 00.364 \end{array}$	$\begin{array}{c} \text{Scaled RMSE} \\ 0.372 \\ 0.362 \\ 2.073 \\ 0.284 \\ 0.238 \\ 1.439 \\ 0.235 \\ 0.190 \\ 1.229 \\ 0.216 \\ 0.216 \\ 0.166 \\ 1.158 \\ 0.182 \end{array}$
n=50 n=100 n=150 n=200 n=250	$ \begin{array}{c} \beta \\ \gamma \\ p \\ \gamma \\ \gamma \\ p \\ \gamma \\ \gamma \\ \gamma \\ \gamma \\ \gamma$	$\begin{array}{r} 3.\\ Mean\\ 02.353\\ 05.134\\ 12.865\\ 02.237\\ 04.984\\ 10.503\\ 02.158\\ 04.953\\ 09.652\\ 02.141\\ 04.987\\ 09.101\\ 02.084\\ 05.043\end{array}$	$\begin{array}{l} \alpha_0 = 1.2, \beta_0 = \\ \hline \text{Relative bias} \\ -0.176 \\ -0.026 \\ -0979 \\ -0.118 \\ 0.033 \\ -0.616 \\ -0.079 \\ 0.009 \\ -0.485 \\ -0.070 \\ 0.003 \\ -0.400 \\ -0.040 \\ -0.042 \\ -0.008 \end{array}$	$\begin{array}{r} 2, \gamma_0 = 5, p_1\\ \hline \text{variance}\\ 000.428\\ 003.262\\ 141.060\\ 000.267\\ 001.414\\ 071.433\\ 000.195\\ 000.9834\\ 000.167\\ 000.691\\ 049.919\\ 000.126\\ 000.608 \end{array}$	$\begin{array}{l} \mathbf{p} = 6.5\\ \mathbf{RMSE}\\ 00.744\\ 01.811\\ 13.475\\ 00.569\\ 01.189\\ 09.352\\ 00.469\\ 00.952\\ 07.985\\ 00.433\\ 00.832\\ 07.529\\ 00.364\\ 00.781 \end{array}$	$\begin{array}{c} \text{Scaled RMSE} \\ 0.372 \\ 0.362 \\ 2.073 \\ 0.284 \\ 0.238 \\ 1.439 \\ 0.235 \\ 0.190 \\ 1.229 \\ 0.216 \\ 0.166 \\ 1.158 \\ 0.182 \\ 0.156 \end{array}$
n=50 n=100 n=150 n=200 n=250	$ \begin{array}{c} \beta \\ \gamma \\ p \\ \gamma \\ p \\ \beta \\ \gamma \\ p \\ \gamma \\ \gamma$	$\begin{array}{r} 3.\\ \text{Mean}\\ 02.353\\ 05.134\\ 12.865\\ 02.237\\ 04.984\\ 10.503\\ 02.158\\ 04.953\\ 09.652\\ 02.141\\ 04.987\\ 09.101\\ 02.084\\ 05.043\\ 08.212 \end{array}$	$\begin{array}{r} \alpha_0 = 1.2, \beta_0 = \\ \hline \text{Relative bias} \\ -0.176 \\ -0.026 \\ -0979 \\ -0.118 \\ 0.033 \\ -0.616 \\ -0.079 \\ 0.009 \\ -0.485 \\ -0.070 \\ 0.003 \\ -0.485 \\ -0.070 \\ 0.003 \\ -0.400 \\ -0.042 \\ -0.008 \\ -0.263 \end{array}$	$\begin{array}{c} 2, \gamma_0 = 5, p \\ \hline \text{variance} \\ 000.428 \\ 003.262 \\ 141.060 \\ 000.267 \\ 001.414 \\ 071.433 \\ 000.195 \\ 000.903 \\ 053.834 \\ 000.167 \\ 000.691 \\ 000.691 \\ 000.126 \\ 000.126 \\ 000.6081 \\ 029.810 \end{array}$	$\begin{array}{l} \mathbf{p} = 6.5 \\ \mathbf{RMSE} \\ 00.744 \\ 01.811 \\ 13.475 \\ 00.569 \\ 01.189 \\ 09.352 \\ 00.469 \\ 00.952 \\ 07.985 \\ 00.433 \\ 00.832 \\ 07.529 \\ 00.364 \\ 00.781 \\ 05.722 \end{array}$	$\begin{array}{c} \text{Scaled RMSE} \\ 0.372 \\ 0.362 \\ 2.073 \\ 0.284 \\ 0.238 \\ 1.439 \\ 0.235 \\ 0.190 \\ 1.229 \\ 0.216 \\ 0.166 \\ 1.158 \\ 0.182 \\ 0.156 \\ 0.880 \\ \end{array}$
n=50 $n=100$ $n=150$ $n=200$ $n=250$ $n=300$	$ \begin{array}{c} \beta \\ \gamma \\ \mathbf{p} \\ \beta \\ \gamma \\ \mathbf{p} \\ \beta \\ \gamma \\ p \\ \gamma \\ p \\ \beta \\ \gamma \\ p \\ \gamma \\ \gamma$	$\begin{array}{c} 3.\\ Mean\\ 02.353\\ 05.134\\ 12.865\\ 02.237\\ 04.984\\ 10.503\\ 02.158\\ 04.953\\ 09.652\\ 02.141\\ 04.987\\ 09.101\\ 02.084\\ 05.043\\ 08.212\\ 02.065\end{array}$	$\begin{array}{r} \alpha_0 = 1.2, \beta_0 = \\ \hline \text{Relative bias} \\ -0.176 \\ -0.026 \\ -0.979 \\ -0.118 \\ 0.033 \\ -0.616 \\ -0.079 \\ 0.009 \\ -0.485 \\ -0.070 \\ 0.003 \\ -0.400 \\ -0.003 \\ -0.400 \\ -0.042 \\ -0.008 \\ -0.263 \\ -0.033 \end{array}$	$\begin{array}{c} 2, \gamma_0 = 5, p_0 \\ \hline \text{variance} \\ 000.428 \\ 003.262 \\ 141.060 \\ 000.267 \\ 001.414 \\ 071.433 \\ 000.195 \\ 000.903 \\ 053.834 \\ 000.167 \\ 000.691 \\ 049.919 \\ 000.126 \\ 000.608 \\ 029.810 \\ 000.092 \end{array}$	$\begin{array}{l} \mathbf{p} = 6.5\\ \mathbf{RMSE}\\ 00.744\\ 01.811\\ 13.475\\ 00.569\\ 01.189\\ 09.352\\ 00.469\\ 00.952\\ 07.985\\ 00.433\\ 00.832\\ 07.529\\ 00.364\\ 00.781\\ 05.722\\ 00.309 \end{array}$	$\begin{array}{c} \text{Scaled RMSE} \\ 0.372 \\ 0.362 \\ 2.073 \\ 0.284 \\ 0.238 \\ 1.439 \\ 0.235 \\ 0.190 \\ 1.229 \\ 0.216 \\ 0.166 \\ 1.158 \\ 0.166 \\ 1.158 \\ 0.156 \\ 0.880 \\ 0.155 \end{array}$
n=50 n=100 n=150 n=200 n=250 n=300	$ \begin{array}{c} \beta \\ \gamma \\ \mathbf{p} \\ \beta \\ \gamma \\ \beta \\ \gamma \\ \mathbf{p} \\ \beta \\ \gamma \\ \gamma \\ \beta \\ \gamma \\ \gamma \\ \beta \\ \gamma \\ \gamma \\ \beta \\ \gamma \\ \gamma$	$\begin{array}{c} 3.\\ Mean\\ 02.353\\ 05.134\\ 12.865\\ 02.237\\ 04.984\\ 10.503\\ 02.158\\ 04.953\\ 09.652\\ 02.141\\ 04.987\\ 09.101\\ 02.084\\ 05.043\\ 08.212\\ 02.065\\ 05.023\\ \end{array}$	$\begin{array}{r} \alpha_0 = 1.2, \beta_0 = \\ \hline \text{Relative bias} \\ -0.176 \\ -0.026 \\ -0979 \\ -0.118 \\ 0.033 \\ -0.616 \\ -0.079 \\ 0.009 \\ -0.485 \\ -0.070 \\ 0.003 \\ -0.400 \\ -0.042 \\ -0.008 \\ -0.263 \\ -0.033 \\ -0.005 \end{array}$	$\begin{array}{r} 2, \gamma_0 = 5, p_0 \\ \hline \text{variance} \\ 000.428 \\ 003.262 \\ 141.060 \\ 000.267 \\ 001.414 \\ 071.433 \\ 000.195 \\ 000.903 \\ 053.834 \\ 000.167 \\ 000.691 \\ 049.919 \\ 000.126 \\ 000.608 \\ 029.810 \\ 000.092 \\ 000.427 \end{array}$	$\begin{array}{l} \mathbf{b} = 6.5 \\ \hline \mathbf{RMSE} \\ 00.744 \\ 01.811 \\ 13.475 \\ 00.569 \\ 01.189 \\ 09.352 \\ 00.469 \\ 00.952 \\ 07.985 \\ 00.433 \\ 00.832 \\ 07.529 \\ 00.364 \\ 00.781 \\ 05.722 \\ 00.309 \\ 00.653 \end{array}$	$\begin{array}{c} \text{Scaled RMSE} \\ 0.372 \\ 0.362 \\ 2.073 \\ 0.284 \\ 0.238 \\ 1.439 \\ 0.235 \\ 0.190 \\ 1.229 \\ 0.216 \\ 0.166 \\ 1.158 \\ 0.182 \\ 0.156 \\ 0.880 \\ 0.155 \\ 0.131 \\ \end{array}$
n=50 n=100 n=150 n=200 n=250 n=300	$ \begin{array}{c} \beta \\ \gamma \\ \mathbf{p} \\ \gamma \\ \mathbf{p} \\ \beta \\ \gamma \\ \mathbf{p} \\ \gamma \\ $	$\begin{array}{r} 3.\\ \text{Mean}\\ 02.353\\ 05.134\\ 12.865\\ 02.237\\ 04.984\\ 10.503\\ 02.158\\ 04.953\\ 09.652\\ 02.141\\ 04.987\\ 09.101\\ 02.084\\ 05.043\\ 08.212\\ 02.065\\ 05.023\\ 07.803\\ \end{array}$	$\begin{array}{r} \alpha_0 = 1.2, \beta_0 = \\ \hline \text{Relative bias} \\ -0.176 \\ -0.026 \\ -0.979 \\ -0.118 \\ 0.033 \\ -0.616 \\ -0.079 \\ 0.009 \\ -0.485 \\ -0.070 \\ 0.0009 \\ -0.485 \\ -0.070 \\ 0.003 \\ -0.400 \\ -0.042 \\ -0.008 \\ -0.263 \\ -0.005 \\ -0.201 \end{array}$	$\begin{array}{c} 2, \gamma_0 = 5, p \\ \hline \text{variance} \\ 000.428 \\ 003.262 \\ 141.060 \\ 000.267 \\ 001.414 \\ 071.433 \\ 000.195 \\ 000.903 \\ 053.834 \\ 000.195 \\ 000.691 \\ 000.691 \\ 000.691 \\ 000.691 \\ 000.608 \\ 029.810 \\ 000.092 \\ 000.427 \\ 018.784 \end{array}$	$\begin{array}{l} \mathbf{p} = 6.5\\ \mathbf{RMSE}\\ 00.744\\ 01.811\\ 13.475\\ 00.569\\ 01.189\\ 09.352\\ 00.469\\ 00.952\\ 07.985\\ 00.433\\ 00.832\\ 07.529\\ 00.364\\ 00.781\\ 05.722\\ 00.309\\ 00.653\\ 04.525 \end{array}$	$\begin{array}{c} \text{Scaled RMSE} \\ 0.372 \\ 0.362 \\ 2.073 \\ 0.284 \\ 0.238 \\ 1.439 \\ 0.235 \\ 0.190 \\ 1.229 \\ 0.216 \\ 0.166 \\ 1.158 \\ 0.182 \\ 0.156 \\ 0.182 \\ 0.156 \\ 0.182 \\ 0.155 \\ 0.131 \\ 0.696 \end{array}$
n=50 $n=100$ $n=150$ $n=200$ $n=250$ $n=300$	$ \begin{array}{c} \beta \\ \gamma \\ p \\ \\ p \\ \gamma \\ p \\ p \\ \gamma \\ p \\ p \\ \gamma \\ p \\ p$	$\begin{array}{c} 3.\\ \text{Mean}\\ 02.353\\ 05.134\\ 12.865\\ 02.237\\ 04.984\\ 10.503\\ 02.158\\ 04.953\\ 09.652\\ 02.141\\ 04.987\\ 09.101\\ 02.084\\ 05.043\\ 08.212\\ 02.065\\ 05.023\\ 07.803\\ 07.803\\ 4. \end{array}$	$\begin{array}{l} \alpha_0 = 1.2, \beta_0 = \\ \hline \text{Relative bias} \\ -0.176 \\ -0.026 \\ -0.979 \\ -0.118 \\ 0.033 \\ -0.616 \\ -0.079 \\ 0.009 \\ -0.485 \\ -0.070 \\ 0.003 \\ -0.400 \\ -0.003 \\ -0.040 \\ -0.042 \\ -0.008 \\ -0.263 \\ -0.263 \\ -0.033 \\ -0.005 \\ -0.201 \\ \hline \alpha_0 = 1.5, \beta_0 = \end{array}$	$\begin{array}{c} 2, \gamma_0 = 5, p_0 \\ \hline \text{variance} \\ 000.428 \\ 003.262 \\ 141.060 \\ 000.267 \\ 001.414 \\ 071.433 \\ 000.195 \\ 000.903 \\ 053.834 \\ 000.167 \\ 000.691 \\ 049.919 \\ 000.126 \\ 000.608 \\ 029.810 \\ 000.092 \\ 000.427 \\ 018.784 \\ 3.5, \gamma_0 = 5, \end{array}$	$\begin{array}{l} \mathbf{p} = 6.5 \\ \mathbf{RMSE} \\ 00.744 \\ 01.811 \\ 13.475 \\ 00.569 \\ 01.189 \\ 09.352 \\ 00.469 \\ 00.952 \\ 07.985 \\ 00.433 \\ 00.832 \\ 07.529 \\ 00.364 \\ 00.781 \\ 05.722 \\ 00.309 \\ 00.653 \\ $	$\begin{array}{c} {\rm Scaled\ RMSE}\\ 0.372\\ 0.362\\ 2.073\\ 0.284\\ 0.238\\ 1.439\\ 0.235\\ 0.190\\ 1.229\\ 0.216\\ 0.166\\ 1.158\\ 0.182\\ 0.156\\ 0.880\\ 0.156\\ 0.880\\ 0.155\\ 0.131\\ 0.696\\ \end{array}$
n=50 n=100 n=150 n=200 n=250 n=300	$ \begin{array}{c} \beta \\ \gamma \\ \mathbf{p} \\ \gamma \\ \mathbf{p} \\ \beta \\ \gamma \\ \mathbf{p} \\ \beta \\ \gamma \\ \mathbf{p} \\ \beta \\ \gamma \\ \mathbf{p} \\ \gamma \\ \mathbf{p} \\ \gamma \\ \mathbf{p} \\ \beta \\ \gamma \\ \gamma$	$\begin{array}{c} 3.\\ \text{Mean}\\ 02.353\\ 05.134\\ 12.865\\ 02.237\\ 04.984\\ 10.503\\ 02.158\\ 04.953\\ 09.652\\ 02.141\\ 04.987\\ 09.101\\ 02.084\\ 05.043\\ 08.212\\ 02.065\\ 05.023\\ 07.803\\ 07.803\\ 4.\\ \text{Mean} \end{array}$	$\begin{array}{l} \alpha_0 = 1.2, \beta_0 = \\ \hline \text{Relative bias} \\ -0.176 \\ -0.026 \\ -0979 \\ -0.118 \\ 0.033 \\ -0.616 \\ -0.079 \\ 0.009 \\ -0.485 \\ -0.070 \\ 0.009 \\ -0.485 \\ -0.070 \\ 0.003 \\ -0.400 \\ -0.042 \\ -0.003 \\ -0.003 \\ -0.003 \\ -0.003 \\ -0.263 \\ -0.033 \\ -0.005 \\ -0.201 \\ \alpha_0 = 1.5, \beta_0 = \\ \hline \text{Relative bias} \end{array}$	$\begin{array}{c} 2, \gamma_0 = 5, p_0 \\ \hline \text{variance} \\ 000.428 \\ 003.262 \\ 141.060 \\ 000.267 \\ 001.414 \\ 071.433 \\ 000.195 \\ 000.903 \\ 053.834 \\ 000.167 \\ 000.691 \\ 000.691 \\ 000.126 \\ 000.608 \\ 029.810 \\ 000.000.928 \\ 000.427 \\ 018.784 \\ 3.5, \gamma_0 = 5, \\ \text{variance} \end{array}$	$\begin{array}{l} \mathbf{p} = 6.5 \\ \mathbf{RMSE} \\ 00.744 \\ 01.811 \\ 13.475 \\ 00.569 \\ 01.189 \\ 09.352 \\ 00.469 \\ 00.952 \\ 00.469 \\ 00.952 \\ 07.985 \\ 00.433 \\ 00.832 \\ 07.529 \\ 00.364 \\ 00.781 \\ 05.722 \\ 00.309 \\ 00.653 \\ 04.525 \\ p_0 = 7 \\ \mathbf{RMSE} \end{array}$	$\begin{array}{c} {\rm Scaled\ RMSE}\\ 0.372\\ 0.362\\ 2.073\\ 0.284\\ 0.238\\ 1.439\\ 0.235\\ 0.190\\ 1.229\\ 0.216\\ 0.166\\ 1.158\\ 0.182\\ 0.156\\ 0.880\\ 0.155\\ 0.131\\ 0.696\\ \end{array}$
n=50 n=100 n=150 n=200 n=250 n=300	$ \begin{array}{c} \beta \\ \gamma \\ p \\ \gamma \\ p \\ \beta \\ \gamma \\ p \\ \gamma \\ p \\ \beta \\ \gamma \\ p \\ \gamma \\ \gamma$	$\begin{array}{c} 3.\\ \text{Mean}\\ 02.353\\ 05.134\\ 12.865\\ 02.237\\ 04.984\\ 10.503\\ 02.158\\ 04.953\\ 09.652\\ 02.141\\ 04.987\\ 09.101\\ 02.084\\ 05.043\\ 08.212\\ 02.065\\ 05.023\\ 07.803\\ 07.803\\ 4.\\ \text{Mean}\\ 04.067\\ \end{array}$	$\begin{array}{l} \alpha_0 = 1.2, \beta_0 = \\ \hline \text{Relative bias} \\ -0.176 \\ -0.026 \\ -0979 \\ -0.118 \\ 0.033 \\ -0.616 \\ -0.079 \\ 0.009 \\ -0.485 \\ -0.070 \\ 0.009 \\ -0.485 \\ -0.070 \\ 0.003 \\ -0.400 \\ -0.042 \\ -0.008 \\ -0.263 \\ -0.005 \\ -0.201 \\ \alpha_0 = 1.5, \beta_0 = \\ \hline \text{Relative bias} \\ -0.162 \end{array}$	$\begin{array}{c} 2, \gamma_0 = 5, p \\ \hline \text{variance} \\ 000.428 \\ 003.262 \\ 141.060 \\ 000.267 \\ 001.414 \\ 071.433 \\ 000.195 \\ 000.903 \\ 053.834 \\ 000.195 \\ 000.691 \\ 000.691 \\ 000.691 \\ 000.691 \\ 000.693 \\ 0000.693 \\ 0000.693 \\ 000.693 \\ 000.693 \\ 000.693 \\ 000.6$	$\begin{array}{l} \mathbf{p} = 6.5 \\ \mathbf{RMSE} \\ 00.744 \\ 01.811 \\ 13.475 \\ 00.569 \\ 01.189 \\ 09.352 \\ 00.469 \\ 00.952 \\ 07.985 \\ 00.433 \\ 00.832 \\ 07.529 \\ 00.364 \\ 00.781 \\ 05.722 \\ 00.364 \\ 00.653 \\ 04.525 \\ \hline p_0 = 7 \\ \mathbf{RMSE} \\ 1.360 \end{array}$	$\begin{array}{c} \text{Scaled RMSE} \\ 0.372 \\ 0.362 \\ 2.073 \\ 0.284 \\ 0.238 \\ 1.439 \\ 0.235 \\ 0.190 \\ 1.229 \\ 0.216 \\ 0.166 \\ 1.158 \\ 0.182 \\ 0.156 \\ 0.182 \\ 0.155 \\ 0.131 \\ 0.696 \\ \hline \\ \text{Scaled RMSE} \\ 0.388 \\ \hline \end{array}$
n=50 $n=100$ $n=150$ $n=200$ $n=250$ $n=300$ $n=50$	$ \begin{array}{c} \beta \\ \gamma \\ p \\ \\ \gamma \\ \gamma$	$\begin{array}{r} 3.\\ Mean\\ 02.353\\ 05.134\\ 12.865\\ 02.237\\ 04.984\\ 10.503\\ 02.158\\ 04.953\\ 09.652\\ 02.141\\ 04.987\\ 09.101\\ 02.084\\ 05.043\\ 08.212\\ 02.065\\ 05.023\\ 07.803\\ 07.803\\ 04.972\end{array}$	$\begin{array}{r} \alpha_0 = 1.2, \beta_0 = \\ \hline \text{Relative bias} \\ -0.176 \\ -0.026 \\ -0979 \\ -0.118 \\ 0.033 \\ -0.616 \\ -0.079 \\ 0.009 \\ -0.485 \\ -0.070 \\ 0.003 \\ -0.485 \\ -0.070 \\ 0.003 \\ -0.040 \\ -0.042 \\ -0.008 \\ -0.263 \\ -0.033 \\ -0.005 \\ -0.201 \\ \hline \alpha_0 = 1.5, \beta_0 = \\ \hline \text{Relative bias} \\ -0.162 \\ 0.005 \\ -0.005 \end{array}$	$\begin{array}{r} 2, \gamma_0 = 5, p_1\\ \hline \text{variance}\\ 000.428\\ 003.262\\ 141.060\\ 000.267\\ 001.414\\ 071.433\\ 000.195\\ 000.95\\ 000.195\\ 000.167\\ 000.691\\ 049.919\\ 000.126\\ 000.608\\ 029.810\\ 000.092\\ 000.427\\ 018.784\\ 3.5, \gamma_0 = 5,\\ \hline \text{variance}\\ 01.528\\ 02.854 \end{array}$	$\begin{array}{l} \mathbf{p} = 6.5 \\ \mathbf{RMSE} \\ 00.744 \\ 01.811 \\ 13.475 \\ 00.569 \\ 01.189 \\ 09.352 \\ 00.469 \\ 00.952 \\ 07.985 \\ 00.433 \\ 00.832 \\ 07.529 \\ 00.364 \\ 00.781 \\ 00.781 \\ 00.781 \\ 00.653 \\ 04.525 \\ \hline p_0 = 7 \\ \mathbf{RMSE} \\ 1.360 \\ 1.689 \end{array}$	$\begin{array}{c} {\rm Scaled\ RMSE}\\ 0.372\\ 0.362\\ 2.073\\ 0.284\\ 0.238\\ 1.439\\ 0.235\\ 0.190\\ 1.229\\ 0.216\\ 0.166\\ 1.158\\ 0.182\\ 0.156\\ 0.880\\ 0.156\\ 0.880\\ 0.155\\ 0.131\\ 0.696\\ \hline \\ {\rm Scaled\ RMSE}\\ 0.338\\ 0.338\\ \hline \end{array}$
n=50 n=100 n=150 n=200 n=250 n=300 n=50	$ \begin{array}{c} \beta \\ \gamma \\ \mathbf{p} \\ \beta \\ \gamma \\ \gamma$	$\begin{array}{r} 3.\\ \text{Mean}\\ 02.353\\ 05.134\\ 12.865\\ 02.237\\ 04.984\\ 10.503\\ 02.158\\ 04.953\\ 09.652\\ 02.141\\ 04.987\\ 09.101\\ 02.084\\ 05.043\\ 08.212\\ 02.065\\ 05.023\\ 07.803\\ 4.\\ \text{Mean}\\ 04.067\\ 04.972\\ 12.800\\ \end{array}$	$\begin{array}{r} \alpha_0 = 1.2, \beta_0 = \\ \hline \text{Relative bias} \\ -0.176 \\ -0.026 \\ -0.979 \\ -0.118 \\ 0.033 \\ -0.616 \\ -0.079 \\ 0.009 \\ -0.485 \\ -0.070 \\ 0.009 \\ -0.485 \\ -0.070 \\ 0.003 \\ -0.485 \\ -0.070 \\ 0.003 \\ -0.263 \\ -0.003 \\ -0.263 \\ -0.005 \\ -0.201 \\ \hline \alpha_0 = 1.5, \beta_0 = \\ \hline \text{Relative bias} \\ -0.162 \\ 0.005 \\ -0.828 \end{array}$	$\begin{array}{r} 2, \gamma_0 = 5, p \\ \hline \text{variance} \\ 000.428 \\ 003.262 \\ 141.060 \\ 000.267 \\ 001.414 \\ 071.433 \\ 000.195 \\ 000.903 \\ 053.834 \\ 000.195 \\ 000.691 \\ 000.126 \\ 000.691 \\ 000.126 \\ 000.691 \\ 000.427 \\ 018.784 \\ 3.5, \gamma_0 = 5, \\ \text{variance} \\ 01.528 \\ 02.854 \\ 93.549 \end{array}$	$\begin{array}{l} \mathbf{p} = 6.5 \\ \mathbf{RMSE} \\ 00.744 \\ 01.811 \\ 13.475 \\ 00.569 \\ 01.189 \\ 09.352 \\ 00.469 \\ 00.952 \\ 07.985 \\ 00.433 \\ 00.832 \\ 07.529 \\ 00.364 \\ 00.781 \\ 05.722 \\ 00.309 \\ 00.653 \\ 04.525 \\ p_0 = 7 \\ \mathbf{RMSE} \\ 1.360 \\ 1.689 \\ 11.278 \end{array}$	$\begin{array}{c} \text{Scaled RMSE} \\ 0.372 \\ 0.362 \\ 2.073 \\ 0.284 \\ 0.238 \\ 1.439 \\ 0.235 \\ 0.190 \\ 1.229 \\ 0.216 \\ 0.166 \\ 1.158 \\ 0.182 \\ 0.156 \\ 0.182 \\ 0.155 \\ 0.182 \\ 0.155 \\ 0.131 \\ 0.696 \\ \hline \\ \text{Scaled RMSE} \\ 0.388 \\ 0.338 \\ 1.611 \\ \hline \end{array}$
n=50 $n=100$ $n=150$ $n=200$ $n=250$ $n=300$ $n=50$ $n=100$	$\begin{array}{c} \beta \\ \gamma \\ \mathbf{p} \\ \\ \gamma \\ \gamma \\ \mathbf{p} \\ \gamma \\ $	$\begin{array}{r} 3.\\ \text{Mean}\\ 02.353\\ 05.134\\ 12.865\\ 02.237\\ 04.984\\ 10.503\\ 02.158\\ 04.953\\ 09.652\\ 02.141\\ 04.987\\ 09.101\\ 02.084\\ 05.043\\ 08.212\\ 02.065\\ 05.023\\ 07.803\\ 07.803\\ 4.\\ \text{Mean}\\ 04.067\\ 04.972\\ 12.800\\ 03.848\end{array}$	$\begin{array}{r} \alpha_0 = 1.2, \beta_0 = \\ \hline \text{Relative bias} \\ -0.176 \\ -0.026 \\ -0.979 \\ -0.118 \\ 0.033 \\ -0.616 \\ -0.079 \\ 0.009 \\ -0.485 \\ -0.070 \\ 0.009 \\ -0.485 \\ -0.070 \\ 0.003 \\ -0.400 \\ -0.042 \\ -0.008 \\ -0.263 \\ -0.005 \\ -0.201 \\ \hline \alpha_0 = 1.5, \beta_0 = \\ \hline \text{Relative bias} \\ -0.162 \\ 0.005 \\ -0.828 \\ -0.099 \end{array}$	$\begin{array}{r} 2, \gamma_0 = 5, p \\ \hline \text{variance} \\ 000.428 \\ 003.262 \\ 141.060 \\ 000.267 \\ 001.414 \\ 071.433 \\ 000.195 \\ 000.903 \\ 053.834 \\ 000.195 \\ 000.691 \\ 000.691 \\ 000.691 \\ 000.691 \\ 000.691 \\ 000.693 \\ 000.491 \\ 000.608 \\ 029.810 \\ 000.427 \\ 018.784 \\ 3.5, \gamma_0 = 5, \\ \hline \text{variance} \\ 01.528 \\ 02.854 \\ 93.549 \\ 00.451 \end{array}$	$\begin{array}{c} \mathbf{b} = 6.5 \\ \mathbf{RMSE} \\ 00.744 \\ 01.811 \\ 13.475 \\ 00.569 \\ 01.189 \\ 09.352 \\ 00.469 \\ 00.952 \\ 07.985 \\ 00.433 \\ 00.832 \\ 07.529 \\ 00.364 \\ 00.781 \\ 05.722 \\ 00.364 \\ 00.781 \\ 05.722 \\ 00.364 \\ 00.781 \\ 0.572 \\ 00.364 \\ 0.759 \\ 0.369 \\ 1.689 \\ 1.278 \\ 0.756 \end{array}$	$\begin{array}{c} {\rm Scaled \ RMSE} \\ 0.372 \\ 0.362 \\ 2.073 \\ 0.238 \\ 1.439 \\ 0.238 \\ 1.439 \\ 0.235 \\ 0.190 \\ 1.229 \\ 0.216 \\ 0.166 \\ 1.158 \\ 0.182 \\ 0.156 \\ 0.182 \\ 0.155 \\ 0.131 \\ 0.696 \\ \hline \\ {\rm Scaled \ RMSE} \\ 0.388 \\ 0.338 \\ 1.611 \\ 0.216 \\ \hline \end{array}$
n=50 $n=100$ $n=150$ $n=200$ $n=250$ $n=300$ $n=50$ $n=100$	$ \begin{array}{c} \beta \\ \gamma \\ p \\ \beta \\ \gamma \\ p \\ p \\ \beta \\ \gamma \\ p \\ p \\ p \\ \beta \\ \gamma \\ p \\ p \\ p \\ \beta \\ \gamma \\ p \\ p \\ \beta \\ \gamma \\ p \\ p \\ \beta \\ \gamma \\ p \\ p \\ p \\ \beta \\ \gamma \\ p \\ p \\ \beta \\ \gamma \\ p \\ p \\ p \\ \beta \\ \gamma \\ p \\ p$	$\begin{array}{r} 3.\\ Mean\\ 02.353\\ 05.134\\ 12.865\\ 02.237\\ 04.984\\ 10.503\\ 02.158\\ 04.953\\ 09.652\\ 02.141\\ 04.987\\ 09.101\\ 02.084\\ 05.043\\ 08.212\\ 02.065\\ 05.023\\ 07.803\\ 04.972\\ 12.800\\ 03.848\\ 04.878\end{array}$	$\begin{array}{r} \alpha_0 = 1.2, \beta_0 = \\ \hline \text{Relative bias} \\ \hline -0.176 \\ -0.026 \\ -0979 \\ \hline -0.118 \\ 0.033 \\ -0.616 \\ -0.079 \\ 0.009 \\ -0.485 \\ \hline -0.070 \\ 0.009 \\ -0.485 \\ \hline -0.070 \\ 0.003 \\ -0.040 \\ -0.042 \\ -0.008 \\ -0.263 \\ -0.033 \\ -0.005 \\ -0.201 \\ \hline \alpha_0 = 1.5, \beta_0 = \\ \hline \text{Relative bias} \\ \hline -0.162 \\ 0.005 \\ -0.828 \\ -0.099 \\ 0.024 \end{array}$	$\begin{array}{r} 2, \gamma_0 = 5, p_1\\ \hline \text{variance}\\ 000.428\\ 003.262\\ 141.060\\ 000.267\\ 001.414\\ 071.433\\ 000.195\\ 000.903\\ 053.834\\ 000.167\\ 000.691\\ 049.919\\ 000.126\\ 000.608\\ 029.810\\ 000.0427\\ 018.784\\ 3.5, \gamma_0 = 5,\\ \text{variance}\\ 01.528\\ 02.854\\ 93.549\\ 00.451\\ 01.212\end{array}$	$\begin{array}{l} \mathbf{p} = 6.5 \\ \mathbf{RMSE} \\ 00.744 \\ 01.811 \\ 13.475 \\ 00.569 \\ 01.189 \\ 09.352 \\ 00.469 \\ 00.952 \\ 07.985 \\ 00.433 \\ 00.832 \\ 07.529 \\ 00.364 \\ 00.781 \\ 05.722 \\ 00.364 \\ 00.781 \\ 05.722 \\ 00.364 \\ 00.756 \\ 1.360 \\ 1.689 \\ 11.278 \\ 0.756 \\ 1.107 \end{array}$	$\begin{array}{c} {\rm Scaled \ RMSE} \\ 0.372 \\ 0.362 \\ 2.073 \\ 0.284 \\ 0.238 \\ 1.439 \\ 0.235 \\ 0.190 \\ 1.229 \\ 0.216 \\ 0.166 \\ 1.158 \\ 0.182 \\ 0.156 \\ 0.182 \\ 0.156 \\ 0.155 \\ 0.131 \\ 0.696 \\ \hline \\ {\rm Scaled \ RMSE} \\ 0.338 \\ 1.611 \\ 0.216 \\ 0.222 \\ \end{array}$
n=50 n=100 n=150 n=200 n=250 n=300 n=50 n=100	$ \begin{array}{c} \beta \\ \gamma \\ p \\ \beta \\ \gamma \\ p \\ p \\ p \\ p \\ \beta \\ \gamma \\ p \\ p$	$\begin{array}{r} 3.\\ \text{Mean}\\ 02.353\\ 05.134\\ 12.865\\ 02.237\\ 04.984\\ 10.503\\ 02.158\\ 04.953\\ 09.652\\ 02.141\\ 04.987\\ 09.101\\ 02.084\\ 05.043\\ 08.212\\ 02.065\\ 05.023\\ 07.803\\ 4.\\ \text{Mean}\\ 04.067\\ 04.972\\ 12.800\\ 03.848\\ 04.878\\ 11.407\\ \end{array}$	$\begin{array}{r} \alpha_0 = 1.2, \beta_0 = \\ \hline \text{Relative bias} \\ -0.176 \\ -0.026 \\ -0.979 \\ -0.118 \\ 0.033 \\ -0.616 \\ -0.079 \\ 0.009 \\ -0.485 \\ -0.070 \\ 0.009 \\ -0.485 \\ -0.070 \\ 0.003 \\ -0.485 \\ -0.070 \\ 0.003 \\ -0.263 \\ -0.033 \\ -0.005 \\ -0.201 \\ \hline \alpha_0 = 1.5, \beta_0 = \\ \hline \text{Relative bias} \\ -0.162 \\ 0.005 \\ -0.828 \\ -0.099 \\ 0.024 \\ -0.629 \end{array}$	$\begin{array}{r} 2, \gamma_0 = 5, p\\ \hline \text{variance}\\ 000.428\\ 003.262\\ 141.060\\ 000.267\\ 001.414\\ 071.433\\ 000.195\\ 000.903\\ 053.834\\ 000.195\\ 000.903\\ 053.834\\ 000.167\\ 000.691\\ 000.126\\ 000.691\\ 000.126\\ 000.691\\ 000.126\\ 000.691\\ 000.427\\ 018.784\\ 3.5, \gamma_0 = 5,\\ \text{variance}\\ 01.528\\ 02.854\\ 93.549\\ 00.451\\ 01.212\\ 53.752 \end{array}$	$\begin{array}{l} \mathbf{p} = 6.5 \\ \mathbf{RMSE} \\ 00.744 \\ 01.811 \\ 13.475 \\ 00.569 \\ 01.189 \\ 09.352 \\ 00.469 \\ 00.952 \\ 00.469 \\ 00.952 \\ 07.985 \\ 00.433 \\ 00.832 \\ 07.529 \\ 00.364 \\ 00.781 \\ 05.722 \\ 00.364 \\ 00.781 \\ 05.722 \\ 00.364 \\ 00.781 \\ 05.722 \\ 00.653 \\ 04.525 \\ p_0 = 7 \\ \mathbf{RMSE} \\ 1.360 \\ 1.689 \\ 11.278 \\ 0.756 \\ 1.107 \\ 8.554 \\ \end{array}$	$\begin{array}{r} {\rm Scaled\ RMSE}\\ 0.372\\ 0.362\\ 2.073\\ 0.284\\ 0.238\\ 1.439\\ 0.235\\ 0.190\\ 1.229\\ 0.216\\ 0.166\\ 1.158\\ 0.182\\ 0.156\\ 0.182\\ 0.156\\ 0.155\\ 0.155\\ 0.131\\ 0.696\\ \hline \\ {\rm Scaled\ RMSE}\\ 0.388\\ 0.338\\ 1.611\\ 0.216\\ 0.222\\ 1.222\\ \hline \end{array}$
n=50 $n=100$ $n=150$ $n=200$ $n=300$ $n=50$ $n=100$ $n=150$	$ \begin{array}{c} \beta \\ \gamma \\ \mathbf{p} \\ \mathbf{p} \\ \beta \\ \gamma \\ \mathbf{p} \\ \beta \\ \beta \\ \gamma \\ \mathbf{p} \\ \beta \\ $	$\begin{array}{r} 3.\\ \text{Mean}\\ 02.353\\ 05.134\\ 12.865\\ 02.237\\ 04.984\\ 10.503\\ 02.158\\ 04.953\\ 09.652\\ 02.141\\ 04.987\\ 09.101\\ 02.084\\ 05.043\\ 08.212\\ 02.065\\ 05.023\\ 07.803\\ \hline \\ 4.\\ \text{Mean}\\ 04.067\\ 04.972\\ 12.800\\ 03.848\\ 04.878\\ 04.878\\ 11.407\\ 03.714\\ \end{array}$	$\begin{array}{r} \alpha_0 = 1.2, \beta_0 = \\ \hline \text{Relative bias} \\ -0.176 \\ -0.026 \\ -0.979 \\ -0.118 \\ 0.033 \\ -0.616 \\ -0.079 \\ 0.009 \\ -0.485 \\ -0.070 \\ 0.009 \\ -0.485 \\ -0.070 \\ 0.003 \\ -0.400 \\ -0.042 \\ -0.008 \\ -0.263 \\ -0.005 \\ -0.201 \\ \hline \alpha_0 = 1.5, \beta_0 = \\ \hline \text{Relative bias} \\ -0.162 \\ 0.005 \\ -0.828 \\ -0.099 \\ 0.024 \\ -0.629 \\ -0.061 \\ \hline \end{array}$	$\begin{array}{r} 2, \gamma_0 = 5, p \\ \hline \text{variance} \\ 000.428 \\ 003.262 \\ 141.060 \\ 000.267 \\ 001.414 \\ 071.433 \\ 000.195 \\ 000.903 \\ 053.834 \\ 000.195 \\ 000.691 \\ 000.691 \\ 000.691 \\ 000.691 \\ 000.691 \\ 000.691 \\ 000.691 \\ 000.693 \\ 000.491 \\ 000.693 \\ 000.491 \\ 000.491 \\ 000.491 \\ 000.492 \\ 000.427 \\ 018.784 \\ 3.5, \gamma_0 = 5, \\ \hline \text{variance} \\ 01.528 \\ 02.854 \\ 93.549 \\ 00.451 \\ 01.212 \\ 53.752 \\ 00.298 \end{array}$	$\begin{array}{l} \mathbf{p} = 6.5 \\ \mathbf{RMSE} \\ 00.744 \\ 01.811 \\ 13.475 \\ 00.569 \\ 01.189 \\ 09.352 \\ 07.985 \\ 00.433 \\ 00.952 \\ 07.985 \\ 00.433 \\ 00.832 \\ 07.529 \\ 00.364 \\ 00.781 \\ 05.722 \\ 00.364 \\ 00.781 \\ 05.722 \\ 00.364 \\ 0.586 \\ 0.756 \\ 1.107 \\ 8.554 \\ 0.586 \end{array}$	$\begin{array}{c} \text{Scaled RMSE} \\ 0.372 \\ 0.362 \\ 2.073 \\ 0.238 \\ 1.439 \\ 0.238 \\ 1.439 \\ 0.235 \\ 0.190 \\ 1.229 \\ 0.216 \\ 0.166 \\ 1.158 \\ 0.182 \\ 0.156 \\ 0.182 \\ 0.155 \\ 0.131 \\ 0.696 \\ \hline \\ \hline \\ \text{Scaled RMSE} \\ 0.388 \\ 0.338 \\ 1.611 \\ 0.216 \\ 0.222 \\ 1.222 \\ 0.167 \\ \hline \end{array}$
n=50 $n=100$ $n=150$ $n=200$ $n=300$ $n=50$ $n=100$ $n=150$	$ \begin{array}{c} \beta \\ \gamma \\ p \\ p \\ \beta \\ \gamma \\ p \\ p \\ \beta \\ \gamma \\ \gamma$	$\begin{array}{r} 3.\\ \text{Mean}\\ 02.353\\ 05.134\\ 12.865\\ 02.237\\ 04.984\\ 10.503\\ 02.158\\ 04.953\\ 09.652\\ 02.141\\ 04.987\\ 09.101\\ 02.084\\ 05.043\\ 08.212\\ 02.065\\ 05.023\\ 07.803\\ \hline & 4.\\ \text{Mean}\\ 04.067\\ 04.972\\ 12.800\\ 03.848\\ 04.878\\ 11.407\\ 03.714\\ 04.951\\ \end{array}$	$\begin{array}{r} \alpha_0 = 1.2, \beta_0 = \\ \hline \text{Relative bias} \\ -0.176 \\ -0.026 \\ -0.979 \\ -0.118 \\ 0.033 \\ -0.616 \\ -0.079 \\ 0.009 \\ -0.485 \\ -0.070 \\ 0.009 \\ -0.485 \\ -0.070 \\ 0.003 \\ -0.263 \\ -0.003 \\ -0.263 \\ -0.003 \\ -0.263 \\ -0.033 \\ -0.005 \\ -0.201 \\ \hline \alpha_0 = 1.5, \beta_0 = \\ \hline \text{Relative bias} \\ -0.162 \\ 0.005 \\ -0.828 \\ -0.099 \\ 0.024 \\ -0.629 \\ -0.061 \\ 0.009 \\ \hline \end{array}$	$\begin{array}{r} 2, \gamma_0 = 5, p_0 \\ \hline \text{variance} \\ 000.428 \\ 003.262 \\ 141.060 \\ 000.267 \\ 001.414 \\ 071.433 \\ 000.195 \\ 000.903 \\ 053.834 \\ \hline 000.167 \\ 000.691 \\ 049.919 \\ 000.126 \\ 000.608 \\ 029.810 \\ 000.092 \\ 000.427 \\ 018.784 \\ 3.5, \gamma_0 = 5, \\ \text{variance} \\ \hline 01.528 \\ 02.854 \\ 93.549 \\ 00.451 \\ 01.212 \\ 53.752 \\ 00.298 \\ 00.872 \\ \end{array}$	$\begin{array}{l} \mathbf{p} = 6.5 \\ \mathbf{RMSE} \\ 00.744 \\ 01.811 \\ 13.475 \\ 00.569 \\ 01.189 \\ 09.352 \\ 00.469 \\ 00.952 \\ 07.985 \\ 00.433 \\ 00.832 \\ 07.529 \\ 00.364 \\ 00.781 \\ 05.722 \\ 00.309 \\ 00.653 \\ 04.525 \\ 00.309 \\ 00.653 \\ 04.525 \\ \mathbf{p}_0 = 7 \\ \mathbf{RMSE} \\ 1.360 \\ 1.689 \\ 11.278 \\ 0.756 \\ 1.107 \\ 8.554 \\ 0.586 \\ 0.935 \\ \end{array}$	$\begin{array}{r} {\rm Scaled \ RMSE} \\ 0.372 \\ 0.362 \\ 2.073 \\ 0.284 \\ 0.238 \\ 1.439 \\ 0.235 \\ 0.190 \\ 1.229 \\ 0.216 \\ 0.166 \\ 1.158 \\ 0.182 \\ 0.156 \\ 0.182 \\ 0.156 \\ 0.182 \\ 0.155 \\ 0.131 \\ 0.696 \\ \hline \\ {\rm Scaled \ RMSE} \\ \hline \\ {\rm Scaled \ RMSE} \\ 0.338 \\ 1.611 \\ 0.216 \\ 0.222 \\ 1.222 \\ 0.167 \\ 0.187 \\ \hline \end{array}$
n=50 $n=100$ $n=150$ $n=200$ $n=250$ $n=300$ $n=50$ $n=100$ $n=150$	$ \begin{array}{c} \beta \\ \gamma \\ \mathbf{p} \\ \mathbf{p} \\ \beta \\ \gamma \\ \gamma \\ \mathbf{p} \\ \beta \\ \gamma \\ \gamma \\ \gamma \\ \beta \\ \gamma \\ \gamma \\ \gamma \\ \gamma \\ \gamma$	$\begin{array}{r} 3.\\ \text{Mean}\\ 02.353\\ 05.134\\ 12.865\\ 02.237\\ 04.984\\ 10.503\\ 02.158\\ 04.953\\ 09.652\\ 02.141\\ 04.987\\ 09.101\\ 02.084\\ 05.043\\ 08.212\\ 02.065\\ 05.023\\ 07.803\\ \hline \\ 4.\\ \text{Mean}\\ 04.067\\ 04.972\\ 12.800\\ 03.848\\ 04.878\\ 11.407\\ 03.714\\ 04.951\\ 09.507\\ \end{array}$	$\begin{array}{r} \alpha_0 = 1.2, \beta_0 = \\ \hline \text{Relative bias} \\ -0.176 \\ -0.026 \\ -0.979 \\ -0.118 \\ 0.033 \\ -0.616 \\ -0.079 \\ 0.009 \\ -0.485 \\ -0.070 \\ 0.009 \\ -0.485 \\ -0.070 \\ 0.003 \\ -0.400 \\ -0.042 \\ -0.003 \\ -0.003 \\ -0.003 \\ -0.263 \\ -0.033 \\ -0.005 \\ -0.201 \\ \hline \alpha_0 = 1.5, \beta_0 = \\ \hline \text{Relative bias} \\ -0.162 \\ 0.005 \\ -0.828 \\ -0.099 \\ 0.024 \\ -0.061 \\ 0.009 \\ -0.358 \\ \hline \end{array}$	$\begin{array}{r} 2, \gamma_0 = 5, p \\ \hline \text{variance} \\ 000.428 \\ 003.262 \\ 141.060 \\ 000.267 \\ 001.414 \\ 071.433 \\ 000.195 \\ 000.903 \\ 053.834 \\ 000.167 \\ 000.691 \\ 000.126 \\ 000.691 \\ 000.126 \\ 000.691 \\ 000.126 \\ 000.691 \\ 000.427 \\ 018.784 \\ 3.5, \gamma_0 = 5, \\ \hline \text{variance} \\ 01.528 \\ 02.854 \\ 93.549 \\ 00.451 \\ 01.212 \\ 53.752 \\ 00.298 \\ 00.872 \\ 27.697 \end{array}$	$\begin{array}{l} \mathbf{p} = 6.5 \\ \mathbf{RMSE} \\ 00.744 \\ 01.811 \\ 13.475 \\ 00.569 \\ 01.189 \\ 09.352 \\ 00.469 \\ 00.952 \\ 00.433 \\ 00.832 \\ 07.985 \\ 00.433 \\ 00.832 \\ 07.529 \\ 00.364 \\ 00.781 \\ 05.722 \\ 00.364 \\ 00.781 \\ 05.722 \\ 00.364 \\ 0.781 \\ 0.653 \\ 04.525 \\ p_0 = 7 \\ \mathbf{RMSE} \\ 1.360 \\ 1.689 \\ 11.278 \\ 0.756 \\ 1.107 \\ 8.554 \\ 0.756 \\ 1.107 \\ 8.554 \\ 0.586 \\ 0.935 \\ 5.829 \end{array}$	$\begin{array}{c} \text{Scaled RMSE} \\ 0.372 \\ 0.362 \\ 2.073 \\ 0.284 \\ 0.238 \\ 1.439 \\ 0.235 \\ 0.190 \\ 1.229 \\ 0.216 \\ 0.166 \\ 1.158 \\ 0.156 \\ 0.156 \\ 0.155 \\ 0.155 \\ 0.155 \\ 0.155 \\ 0.156 \\ 0.388 \\ 0.338 \\ 1.611 \\ 0.216 \\ 0.222 \\ 1.222 \\ 0.167 \\ 0.187 \\ 0.833 \\ \end{array}$
n=50 $n=100$ $n=150$ $n=200$ $n=300$ $n=50$ $n=100$ $n=150$ $n=200$	$ \begin{array}{c} \beta \\ \gamma \\ p \\ \gamma \\ p \\ \beta \\ \gamma \\ p \\ \gamma \\ p \\ \beta \\ \gamma \\ p \\ \gamma \\ \gamma$	$\begin{array}{r} 3.\\ \text{Mean}\\ 02.353\\ 05.134\\ 12.865\\ 02.237\\ 04.984\\ 10.503\\ 02.158\\ 04.953\\ 09.652\\ 02.141\\ 04.987\\ 09.101\\ 02.084\\ 05.043\\ 08.212\\ 02.065\\ 05.023\\ 07.803\\ 03.212\\ 02.065\\ 05.023\\ 07.803\\ 04.878\\ 11.407\\ 04.972\\ 12.800\\ 03.848\\ 04.878\\ 11.407\\ 03.714\\ 04.951\\ 09.507\\ 03.692\\ \end{array}$	$\begin{array}{r} \alpha_0 = 1.2, \beta_0 = \\ \hline \text{Relative bias} \\ -0.176 \\ -0.026 \\ -0979 \\ -0.118 \\ 0.033 \\ -0.616 \\ -0.079 \\ 0.009 \\ -0.485 \\ -0.070 \\ 0.003 \\ -0.485 \\ -0.070 \\ 0.003 \\ -0.040 \\ -0.042 \\ -0.008 \\ -0.263 \\ -0.033 \\ -0.033 \\ -0.005 \\ -0.201 \\ \hline \alpha_0 = 1.5, \beta_0 = \\ \hline \text{Relative bias} \\ -0.162 \\ 0.005 \\ -0.828 \\ -0.099 \\ 0.024 \\ -0.629 \\ -0.061 \\ 0.009 \\ -0.358 \\ -0.055 \\ \hline \end{array}$	$\begin{array}{r} 2, \gamma_0 = 5, p \\ \hline \text{variance} \\ 000.428 \\ 003.262 \\ 141.060 \\ 000.267 \\ 001.414 \\ 071.433 \\ 000.195 \\ 000.091 \\ 000.195 \\ 000.691 \\ 049.919 \\ 000.126 \\ 000.691 \\ 049.919 \\ 000.126 \\ 000.691 \\ 000.92 \\ 000.427 \\ 018.784 \\ 03.5, \gamma_0 = 5, \\ \hline \text{variance} \\ 01.528 \\ 02.854 \\ 93.549 \\ 00.451 \\ 01.212 \\ 53.752 \\ 00.298 \\ 00.872 \\ 27.697 \\ 00.232 \end{array}$	$\begin{array}{l} \mathbf{p} = 6.5 \\ \mathbf{RMSE} \\ 00.744 \\ 01.811 \\ 13.475 \\ 00.569 \\ 01.189 \\ 00.352 \\ 00.469 \\ 00.952 \\ 07.985 \\ 00.433 \\ 00.832 \\ 07.529 \\ 00.364 \\ 00.781 \\ 00.781 \\ 00.781 \\ 00.653 \\ 04.525 \\ 00.309 \\ 00.653 \\ 04.525 \\ \mathbf{p}_0 = 7 \\ \mathbf{RMSE} \\ 1.360 \\ 1.689 \\ 11.278 \\ 0.756 \\ 1.107 \\ 8.554 \\ 0.756 \\ 1.107 \\ 8.554 \\ 0.935 \\ 5.829 \\ 0.518 \\ \end{array}$	$\begin{array}{r} {\rm Scaled \ RMSE} \\ \hline 0.372 \\ 0.362 \\ 2.073 \\ 0.284 \\ 0.238 \\ 1.439 \\ 0.235 \\ 0.190 \\ 1.229 \\ 0.216 \\ 0.316 \\ 0.166 \\ 1.158 \\ 0.182 \\ 0.156 \\ 0.880 \\ 0.156 \\ 0.880 \\ 0.155 \\ 0.131 \\ 0.696 \\ \hline \\ {\rm Scaled \ RMSE} \\ 0.338 \\ 1.611 \\ 0.222 \\ 1.222 \\ 0.167 \\ 0.187 \\ 0.833 \\ 0.148 \\ \hline \end{array}$
n=50 $n=100$ $n=150$ $n=200$ $n=300$ $n=300$ $n=100$ $n=150$ $n=200$	$ \begin{array}{c} \beta \\ \gamma \\ \mathbf{p} \\ \beta \\ \gamma \\ \mathbf{p} \\ \beta \\ \gamma \\ p \\ \gamma \\ \gamma$	$\begin{array}{r} 3.\\ \text{Mean}\\ 02.353\\ 05.134\\ 12.865\\ 02.237\\ 04.984\\ 10.503\\ 02.158\\ 04.953\\ 09.652\\ 02.141\\ 04.987\\ 09.101\\ 02.084\\ 05.043\\ 08.212\\ 02.065\\ 05.023\\ 07.803\\ \hline \\ 4.\\ 04.967\\ 04.972\\ 12.800\\ 03.848\\ 04.878\\ 11.407\\ 03.714\\ 04.951\\ 09.507\\ 03.692\\ 04.894\\ \end{array}$	$\begin{array}{r} \alpha_0 = 1.2, \beta_0 = \\ \hline \text{Relative bias} \\ -0.176 \\ -0.026 \\ -0.979 \\ -0.118 \\ 0.033 \\ -0.616 \\ -0.079 \\ 0.009 \\ -0.485 \\ -0.070 \\ 0.009 \\ -0.485 \\ -0.070 \\ 0.003 \\ -0.400 \\ -0.042 \\ -0.003 \\ -0.003 \\ -0.263 \\ -0.033 \\ -0.005 \\ -0.201 \\ \hline \alpha_0 = 1.5, \beta_0 = \\ \hline \text{Relative bias} \\ -0.162 \\ 0.005 \\ -0.828 \\ -0.024 \\ -0.061 \\ 0.009 \\ -0.358 \\ -0.055 \\ 0.021 \\ \end{array}$	$\begin{array}{r} 2, \gamma_0 = 5, p_0 \\ \hline \text{variance} \\ 000.428 \\ 003.262 \\ 141.060 \\ 000.267 \\ 001.414 \\ 071.433 \\ 000.195 \\ 000.903 \\ 053.834 \\ 000.167 \\ 000.691 \\ 049.919 \\ 000.126 \\ 000.608 \\ 029.810 \\ 000.092 \\ 000.427 \\ 018.784 \\ 3.5, \gamma_0 = 5, \\ \text{variance} \\ 01.528 \\ 02.854 \\ 93.549 \\ 00.451 \\ 01.212 \\ 53.752 \\ 00.285 \\ 93.549 \\ 00.451 \\ 01.212 \\ 53.752 \\ 00.298 \\ 00.872 \\ 27.697 \\ 00.232 \\ 00.725 \end{array}$	$\begin{array}{l} \mathbf{p} = 6.5 \\ \mathbf{RMSE} \\ 00.744 \\ 01.811 \\ 13.475 \\ 00.569 \\ 01.189 \\ 09.352 \\ 00.469 \\ 00.952 \\ 07.985 \\ 00.433 \\ 00.832 \\ 07.529 \\ 00.364 \\ 00.781 \\ 05.722 \\ 00.309 \\ 00.653 \\ 04.525 \\ 00.309 \\ 00.653 \\ 04.525 \\ 00.309 \\ 11.278 \\ 0.756 \\ 1.360 \\ 1.689 \\ 11.278 \\ 0.756 \\ 1.107 \\ 8.554 \\ 0.586 \\ 0.935 \\ 5.829 \\ 0.518 \\ 0.857 \\ \end{array}$	$\begin{array}{r} {\rm Scaled \ RMSE} \\ 0.372 \\ 0.362 \\ 2.073 \\ 0.284 \\ 0.238 \\ 1.439 \\ 0.235 \\ 0.190 \\ 1.229 \\ 0.216 \\ 0.166 \\ 1.158 \\ 0.182 \\ 0.156 \\ 0.182 \\ 0.156 \\ 0.182 \\ 0.156 \\ 0.388 \\ 0.338 \\ 1.611 \\ 0.216 \\ 0.328 \\ 0.338 \\ 1.611 \\ 0.216 \\ 0.222 \\ 1.222 \\ 0.167 \\ 0.187 \\ 0.833 \\ 0.148 \\ 0.172 \\ \end{array}$
n=50 $n=100$ $n=150$ $n=200$ $n=300$ $n=50$ $n=100$ $n=150$ $n=200$	$ \begin{array}{c} \beta \\ \gamma \\ \mathbf{p} \\ \mathbf{p} \\ \beta \\ \gamma \\ \gamma \\ \mathbf{p} \\ \beta \\ \gamma \\ \gamma \\ \mathbf{p} \\ \beta \\ \gamma \\ \gamma$	$\begin{array}{r} 3.\\ \text{Mean}\\ 02.353\\ 05.134\\ 12.865\\ 02.237\\ 04.984\\ 10.503\\ 02.158\\ 04.953\\ 09.652\\ 02.141\\ 04.987\\ 09.101\\ 02.084\\ 05.043\\ 08.212\\ 02.065\\ 05.023\\ 07.803\\ \hline \\ 4.\\ \text{Mean}\\ 04.067\\ 04.972\\ 12.800\\ 03.848\\ 04.878\\ 11.407\\ 03.714\\ 04.951\\ 09.507\\ 03.692\\ 04.894\\ 09.726\end{array}$	$\begin{array}{r} \alpha_0 = 1.2, \beta_0 = \\ \hline \text{Relative bias} \\ -0.176 \\ -0.026 \\ -0.979 \\ -0.118 \\ 0.033 \\ -0.616 \\ -0.079 \\ 0.009 \\ -0.485 \\ -0.070 \\ 0.009 \\ -0.485 \\ -0.070 \\ 0.003 \\ -0.400 \\ -0.042 \\ -0.003 \\ -0.042 \\ -0.003 \\ -0.005 \\ -0.263 \\ -0.005 \\ -0.201 \\ \hline \alpha_0 = 1.5, \beta_0 = \\ \hline \text{Relative bias} \\ -0.162 \\ 0.005 \\ -0.828 \\ -0.099 \\ 0.024 \\ -0.0629 \\ -0.061 \\ 0.009 \\ -0.358 \\ -0.055 \\ 0.021 \\ -0.389 \\ -0.93 \\ -0.389 \\ -0.93 \\ $	$\begin{array}{r} 2, \gamma_0 = 5, p \\ \hline \text{variance} \\ 000.428 \\ 003.262 \\ 141.060 \\ 000.267 \\ 001.414 \\ 071.433 \\ 000.195 \\ 000.903 \\ 053.834 \\ 000.195 \\ 000.903 \\ 053.834 \\ 000.195 \\ 000.903 \\ 000.903 \\ 000.903 \\ 000.903 \\ 000.903 \\ 000.903 \\ 000.903 \\ 000.903 \\ 000.9126 \\ 000.691 \\ 000.691 \\ 000.126 \\ 000.691 \\ 000.691 \\ 000.691 \\ 000.691 \\ 000.691 \\ 000.691 \\ 000.691 \\ 000.691 \\ 000.691 \\ 000.691 \\ 000.691 \\ 000.691 \\ 000.691 \\ 000.691 \\ 000.691 \\ 000.691 \\ 000.691 \\ 000.427 \\ 000.425 \\ 000.451 \\ 01.528 \\ 00.872 \\ 00.725 \\ 00.232 \\ 00.725 \\ 29.232 \end{array}$	$\begin{array}{c} \mathbf{p} = 6.5 \\ \mathbf{RMSE} \\ 00.744 \\ 01.811 \\ 13.475 \\ 00.569 \\ 01.189 \\ 09.352 \\ 00.469 \\ 00.952 \\ 00.433 \\ 00.832 \\ 07.985 \\ 00.433 \\ 00.832 \\ 07.529 \\ 00.364 \\ 00.781 \\ 05.722 \\ 00.364 \\ 00.781 \\ 05.722 \\ 00.364 \\ 1.05.722 \\ 00.364 \\ 0.555 \\ 0.555 \\ 0.935 \\ 5.829 \\ 0.518 \\ 0.857 \\ 6.055 \\ 0.555 \\$	$\begin{array}{r} \text{Scaled RMSE} \\ 0.372 \\ 0.362 \\ 2.073 \\ 0.284 \\ 0.238 \\ 1.439 \\ 0.235 \\ 0.190 \\ 1.229 \\ 0.216 \\ 0.166 \\ 1.158 \\ 0.182 \\ 0.155 \\ 0.182 \\ 0.155 \\ 0.155 \\ 0.155 \\ 0.131 \\ 0.696 \\ \hline \\ \text{Scaled RMSE} \\ 0.388 \\ 0.338 \\ 1.611 \\ 0.216 \\ 0.222 \\ 1.222 \\ 0.167 \\ 0.1222 \\ 0.187 \\ 0.833 \\ 0.148 \\ 0.172 \\ 0.865 \\ \hline \end{array}$
n=50 $n=100$ $n=150$ $n=200$ $n=300$ $n=50$ $n=100$ $n=150$ $n=200$ $n=250$	$ \begin{array}{c} \beta \\ \gamma \\ p \\ \gamma \\ p \\ \beta \\ \gamma \\ p \\ \gamma \\ \gamma$	$\begin{array}{r} 3.\\ Mean\\ 02.353\\ 05.134\\ 12.865\\ 02.237\\ 04.984\\ 10.503\\ 02.158\\ 04.953\\ 09.652\\ 02.141\\ 04.987\\ 09.101\\ 02.084\\ 05.043\\ 08.212\\ 02.065\\ 05.023\\ 07.803\\ 04.878\\ 11.407\\ 04.972\\ 12.800\\ 03.848\\ 04.878\\ 11.407\\ 03.714\\ 04.951\\ 09.507\\ 03.692\\ 04.894\\ 09.726\\ 03.651\\ \end{array}$	$\begin{array}{r} \alpha_0 = 1.2, \beta_0 = \\ \hline \text{Relative bias} \\ \hline -0.176 \\ -0.026 \\ -0979 \\ \hline -0.118 \\ 0.033 \\ -0.616 \\ -0.079 \\ 0.009 \\ -0.485 \\ \hline -0.070 \\ 0.003 \\ -0.485 \\ \hline -0.070 \\ 0.003 \\ -0.003 \\ -0.003 \\ -0.008 \\ -0.263 \\ -0.033 \\ -0.005 \\ -0.201 \\ \hline \alpha_0 = 1.5, \beta_0 = \\ \hline \text{Relative bias} \\ \hline -0.162 \\ 0.005 \\ -0.828 \\ -0.099 \\ 0.024 \\ -0.629 \\ -0.061 \\ 0.009 \\ -0.358 \\ -0.091 \\ -0.389 \\ -0.043 \\ \hline -0.043 \\ -0.043 \\ \hline \end{array}$	$\begin{array}{r} 2, \gamma_0 = 5, p\\ \hline \text{variance}\\ 000.428\\ 003.262\\ 141.060\\ 000.267\\ 001.414\\ 071.433\\ 000.195\\ 000.983\\ 000.195\\ 000.691\\ 049.919\\ 000.126\\ 000.691\\ 049.919\\ 000.126\\ 000.608\\ 029.810\\ 000.092\\ 000.427\\ 018.784\\ 3.5, \gamma_0 = 5,\\ \text{variance}\\ 01.528\\ 02.854\\ 93.549\\ 00.451\\ 01.212\\ 53.752\\ 00.298\\ 00.451\\ 01.212\\ 53.752\\ 00.298\\ 00.451\\ 01.212\\ 53.752\\ 00.298\\ 00.451\\ 01.212\\ 53.752\\ 00.298\\ 00.451\\ 01.212\\ 53.752\\ 00.298\\ 00.451\\ 01.212\\ 53.752\\ 00.298\\ 00.451\\ 01.212\\ 53.752\\ 00.298\\ 00.451\\ 01.212\\ 53.752\\ 00.298\\ 00.451\\ 01.212\\ 53.752\\ 00.298\\ 00.451\\ 01.212\\ 53.752\\ 00.298\\ 00.451\\ 01.212\\ 53.752\\ 00.298\\ 00.661\\ 00.008\\ 0$	$\begin{array}{l} \mathbf{p} = 6.5 \\ \mathbf{RMSE} \\ 00.744 \\ 01.811 \\ 13.475 \\ 00.569 \\ 01.189 \\ 00.9352 \\ 00.469 \\ 00.952 \\ 07.985 \\ 00.433 \\ 00.832 \\ 07.529 \\ 00.364 \\ 00.752 \\ 00.364 \\ 00.781 \\ 00.653 \\ 04.525 \\ 00.309 \\ 00.653 \\ 04.525 \\ 0.756 \\ 1.360 \\ 1.360 \\ 1.360 \\ 1.360 \\ 1.360 \\ 1.369 \\ 11.278 \\ 0.756 \\ 1.107 \\ 8.554 \\ 0.935 \\ 5.829 \\ 0.518 \\ 0.857 \\ 6.055 \\ 0.436 \\ 0.555 \\ 0.436 \\ 0.436 \\ 0.436 \\ 0.436 \\ 0.55 \\ 0.436 \\ 0.436 \\ 0.55 \\ 0.436 \\ 0.436 \\ 0.55 \\ 0.55 \\ 0.436 \\ 0.55 \\$	$\begin{array}{c} {\rm Scaled\ RMSE}\\ \hline 0.372\\ 0.362\\ 2.073\\ 0.284\\ 0.238\\ 1.439\\ 0.235\\ 0.190\\ 1.229\\ 0.216\\ 0.35\\ 0.190\\ 1.229\\ 0.216\\ 0.166\\ 1.158\\ 0.182\\ 0.156\\ 0.182\\ 0.156\\ 0.182\\ 0.156\\ 0.388\\ 0.156\\ 0.338\\ 1.611\\ 0.696\\ \hline \end{array}$
n=50 $n=100$ $n=150$ $n=200$ $n=300$ $n=300$ $n=100$ $n=150$ $n=200$ $n=250$	$ \begin{array}{c} \beta \\ \gamma \\ \mathbf{p} \\ \mathbf{p} \\ \beta \\ \gamma \\ \gamma \\ \beta \\ \gamma \\ \gamma \\ \beta \\ \gamma \\ \gamma \\ \gamma$	$\begin{array}{r} 3.\\ \text{Mean}\\ 02.353\\ 05.134\\ 12.865\\ 02.237\\ 04.984\\ 10.503\\ 02.158\\ 04.953\\ 09.652\\ 02.141\\ 04.987\\ 09.101\\ 02.084\\ 05.043\\ 08.212\\ 02.065\\ 05.023\\ 07.803\\ 08.212\\ 02.065\\ 05.023\\ 07.803\\ 04.972\\ 12.800\\ 03.848\\ 04.878\\ 11.407\\ 03.848\\ 04.972\\ 12.800\\ 03.848\\ 04.972\\ 03.848\\ 04.972\\ 03.848\\ 04.972\\ 03.848\\ 04.972\\ 03.651\\ 04.894\\ 09.726\\ 03.651\\ 04.941\\ 09.577\\ 03.651\\ 04.941\\ 04.951\\ 04$	$\begin{array}{r} \alpha_0 = 1.2, \beta_0 = \\ \hline \text{Relative bias} \\ -0.176 \\ -0.026 \\ -0979 \\ -0.118 \\ 0.033 \\ -0.616 \\ -0.079 \\ 0.009 \\ -0.485 \\ -0.070 \\ 0.009 \\ -0.485 \\ -0.070 \\ 0.003 \\ -0.400 \\ -0.042 \\ -0.003 \\ -0.042 \\ -0.003 \\ -0.003 \\ -0.263 \\ -0.033 \\ -0.005 \\ -0.263 \\ -0.005 \\ -0.261 \\ 0.005 \\ -0.263 \\ -0.005 \\ -0.263 \\ -0.005 \\ -0.261 \\ 0.005 \\ -0.828 \\ -0.061 \\ 0.009 \\ -0.358 \\ -0.055 \\ 0.021 \\ -0.389 \\ -0.043 \\ 0.012 \\ 0.0012 \\ 0.0012 \\ 0.0012 \\ 0.0012 \\ 0.0012 \\ 0.001 \\ 0.0012 \\ 0.001 \\ 0.001 \\ 0.0012 \\ 0.001 \\ 0.000 \\ 0.001 \\ 0.000 \\ 0.001 \\ 0.000 \\ $	$\begin{array}{r} 2, \gamma_0 = 5, p\\ \hline \text{variance}\\ 000.428\\ 003.262\\ 141.060\\ 000.267\\ 001.414\\ 071.433\\ 000.195\\ 000.903\\ 053.834\\ 000.167\\ 000.691\\ 000.691\\ 000.126\\ 000.608\\ 029.810\\ 000.028\\ 000.427\\ 018.784\\ 3.5, \gamma_0 = 5,\\ \hline \text{variance}\\ 01.528\\ 02.854\\ 93.549\\ 00.451\\ 01.212\\ 53.752\\ 00.451\\ 01.212\\ 53.752\\ 00.2854\\ 93.549\\ 00.451\\ 01.212\\ 53.752\\ 00.2854\\ 93.549\\ 00.451\\ 01.212\\ 53.752\\ 00.298\\ 00.872\\ 27.697\\ 00.232\\ 00.725\\ 29.232\\ 00.766\\ 00.567\\ 10.544\\ 00.567\\$	$\begin{array}{l} \mathbf{p} = 6.5 \\ \mathbf{RMSE} \\ 00.744 \\ 01.811 \\ 13.475 \\ 00.569 \\ 01.189 \\ 09.352 \\ 00.469 \\ 00.952 \\ 07.985 \\ 00.433 \\ 00.832 \\ 07.529 \\ 00.364 \\ 00.781 \\ 05.722 \\ 00.309 \\ 00.653 \\ 04.525 \\ p_0 = 7 \\ \mathbf{RMSE} \\ 1.360 \\ 1.689 \\ 11.278 \\ 0.756 \\ 1.107 \\ 8.554 \\ 0.586 \\ 0.935 \\ 5.829 \\ 0.518 \\ 0.857 \\ 6.055 \\ 0.436 \\ 0.756 \\ 0.756 \\ 0.436 \\ 0.756 \\ 0.756 \\ 0.518 \\ 0.857 \\ 6.055 \\ 0.436 \\ 0.756 \\ 0.756 \\ 0.756 \\ 0.756 \\ 0.518 \\ 0.857 \\ 0.635 \\ 0.436 \\ 0.756 \\ 0$	$\begin{array}{c} {\rm Scaled\ RMSE}\\ 0.372\\ 0.362\\ 2.073\\ 0.284\\ 0.238\\ 1.439\\ 0.235\\ 0.190\\ 1.229\\ 0.216\\ 0.166\\ 1.158\\ 0.182\\ 0.166\\ 1.158\\ 0.182\\ 0.156\\ 0.880\\ 0.155\\ 0.131\\ 0.696\\ \hline \\ {\rm Scaled\ RMSE}\\ 0.338\\ 0.338\\ 1.611\\ 0.216\\ 0.222\\ 1.222\\ 1.222\\ 0.167\\ 0.833\\ 0.148\\ 0.388\\ 0.338\\ 1.611\\ 0.216\\ 0.216\\ 0.222\\ 1.222\\ 0.167\\ 0.151\\ 0.833\\ 0.148\\ 0.172\\ 0.865\\ 0.151\\ 0.151\\ 0.55\\ 0.55\\ 0.5$
n=50 $n=100$ $n=150$ $n=200$ $n=300$ $n=300$ $n=100$ $n=150$ $n=200$ $n=250$ $n=250$	$ \begin{array}{c} \beta \\ \gamma \\ p \\ p \\ \gamma \\ p \\ \beta \\ \gamma \\ p \\ p \\ \gamma \\ p \\ \beta \\ \gamma \\ p \\ p \\ \gamma \\ p \\ \beta \\ \gamma \\ p \\ p \\ \gamma \\ \gamma$	$\begin{array}{r} 3.\\ \text{Mean}\\ 02.353\\ 05.134\\ 12.865\\ 02.237\\ 04.984\\ 10.503\\ 02.158\\ 04.953\\ 09.652\\ 02.141\\ 04.987\\ 09.101\\ 02.084\\ 05.043\\ 08.212\\ 02.065\\ 05.023\\ 07.803\\ 4.\\ \text{Mean}\\ 04.067\\ 04.972\\ 12.800\\ 03.848\\ 04.878\\ 11.407\\ 03.714\\ 04.951\\ 09.507\\ 03.692\\ 04.894\\ 09.726\\ 03.651\\ 04.941\\ 04.941\\ 04.941\\ 04.941\\ 08.947\\ \end{array}$	$\begin{array}{r} \alpha_0 = 1.2, \beta_0 = \\ \hline \text{Relative bias} \\ -0.176 \\ -0.026 \\ -0979 \\ -0.118 \\ 0.033 \\ -0.616 \\ -0.079 \\ 0.009 \\ -0.485 \\ -0.070 \\ 0.009 \\ -0.485 \\ -0.070 \\ 0.003 \\ -0.485 \\ -0.070 \\ 0.003 \\ -0.042 \\ -0.008 \\ -0.263 \\ -0.033 \\ -0.005 \\ -0.201 \\ \hline \alpha_0 = 1.5, \beta_0 = \\ \hline \text{Relative bias} \\ -0.162 \\ 0.005 \\ -0.828 \\ -0.099 \\ 0.024 \\ -0.629 \\ -0.629 \\ -0.629 \\ -0.061 \\ 0.009 \\ -0.358 \\ -0.055 \\ 0.021 \\ -0.389 \\ -0.043 \\ 0.012 \\ -0.278 \\ -0.25 \\ $	$\begin{array}{r} 2, \gamma_0 = 5, p \\ \hline \text{variance} \\ 000.428 \\ 003.262 \\ 141.060 \\ 000.267 \\ 001.414 \\ 071.433 \\ 000.195 \\ 000.903 \\ 053.834 \\ 000.195 \\ 000.903 \\ 053.834 \\ 000.195 \\ 000.903 \\ 053.834 \\ 000.195 \\ 000.903 \\ 000.903 \\ 000.903 \\ 000.912 \\ 000.691 \\ 000.691 \\ 000.691 \\ 000.691 \\ 000.691 \\ 000.691 \\ 000.691 \\ 000.691 \\ 000.691 \\ 000.691 \\ 000.691 \\ 000.427 \\ 018.784 \\ 3.5, \gamma_0 = 5, \\ \hline \text{variance} \\ 01.528 \\ 02.854 \\ 00.451 \\ 01.528 \\ 02.854 \\ 93.549 \\ 00.451 \\ 01.528 \\ 00.451 \\ 01.528 \\ 00.872 \\ 23.697 \\ 000.232 \\ 00.725 \\ 29.232 \\ 00.166 \\ 00.567 \\ 17.054 \\ 00.567 \\ 17.054 \end{array}$	$\begin{array}{l} \mathbf{p} = 6.5 \\ \mathbf{RMSE} \\ 00.744 \\ 01.811 \\ 13.475 \\ 00.569 \\ 01.189 \\ 09.352 \\ 00.469 \\ 00.952 \\ 07.985 \\ 00.433 \\ 00.832 \\ 07.529 \\ 00.364 \\ 00.781 \\ 05.722 \\ 00.364 \\ 00.781 \\ 05.722 \\ 00.364 \\ 00.781 \\ 0.552 \\ 0.364 \\ 0.756 \\ 1.107 \\ 8.554 \\ 0.556 \\ 0.935 \\ 5.829 \\ 0.518 \\ 0.857 \\ 6.055 \\ 0.436 \\ 0.756 \\ 4.565 \\ 0.756 \\ 4.566 \\ 0.756 \\ 4.566 \\ 0.756 \\ 4.566 \\ 0.756 \\ 4.566 \\ 0.756 \\ 4.566 \\ 0.756 \\ 0.756 \\ 4.566 \\ 0.756 \\ 0.756 \\ 4.566 \\ 0.75$	$\begin{array}{c} \text{Scaled RMSE} \\ 0.372 \\ 0.362 \\ 2.073 \\ 0.284 \\ 0.238 \\ 1.439 \\ 0.235 \\ 0.190 \\ 1.229 \\ 0.216 \\ 0.166 \\ 1.158 \\ 0.182 \\ 0.156 \\ 0.388 \\ 0.356 \\ 0.155 \\ 0.131 \\ 0.696 \\ \hline \\ \hline \\ \text{Scaled RMSE} \\ 0.388 \\ 0.338 \\ 1.611 \\ 0.216 \\ 0.222 \\ 1.222 \\ 0.167 \\ 0.187 \\ 0.833 \\ 0.148 \\ 0.172 \\ 0.865 \\ 0.125 \\ 0.151 \\ 0.652 \\ 0.151 \\ 0.652 \\ 0.151 \\ 0.652 \\ 0.151 \\ 0.652 \\ 0.151 \\ 0.652 \\ 0.151 \\ 0.652 \\ 0.155 \\ 0.155 \\ 0.652 \\ 0.155 \\ 0.155 \\ 0.652 \\ 0.155 \\ 0.155 \\ 0.652 \\ 0.155 \\ 0.155 \\ 0.652 \\ 0.155 \\ 0.155 \\ 0.155 \\ 0.652 \\ 0.155 \\ 0.155 \\ 0.652 \\ 0.155 \\ 0.155 \\ 0.652 \\ 0.155 \\ 0.155 \\ 0.652 \\ 0.155$
$ \begin{array}{c} \\ n=50 \\ n=100 \\ n=150 \\ n=200 \\ n=250 \\ n=300 \\ \hline \\ n=50 \\ n=100 \\ n=150 \\ n=200 \\ n=250 \\ n=300 \\ \end{array} $	$ \begin{array}{c} \beta \\ \gamma \\ \mathbf{p} \\ \beta \\ \gamma \\ \gamma \\ \mathbf{p} \\ \beta \\ \gamma \\ \gamma \\ \mathbf{p} \\ \beta \\ \gamma \\ \gamma \\ \beta \\ \gamma \\ \gamma \\ \beta \\ \gamma \\ \gamma \\ \gamma$	$\begin{array}{r} 3.\\ Mean\\ 02.353\\ 05.134\\ 12.865\\ 02.237\\ 04.984\\ 10.503\\ 02.158\\ 04.953\\ 09.652\\ 02.141\\ 04.987\\ 09.101\\ 02.084\\ 05.043\\ 08.212\\ 02.065\\ 05.023\\ 07.803\\ 04.951\\ 04.972\\ 12.800\\ 03.848\\ 04.878\\ 11.407\\ 03.714\\ 04.951\\ 09.507\\ 03.692\\ 04.894\\ 09.726\\ 03.651\\ 04.941\\ 08.947\\ 03.625\\ 04.957\\ 04.957\\ 03.625\\ 04.957\\ 04.957\\ 03.625\\ 04.957\\ 04.957\\ 03.625\\ 04.957\\ 04.957\\ 03.625\\ 04.957\\ 04.957\\ 03.625\\ 04.957\\ 04.957\\ 03.625\\ 04.957\\ 04.957\\ 03.625\\ 04.957\\ 04.957\\ 04.957\\ 03.625\\ 04.957\\ 04.957\\ 04.957\\ 04.957\\ 03.625\\ 04.957\\ 04.95$	$\begin{array}{r} \alpha_0 = 1.2, \beta_0 = \\ \hline \text{Relative bias} \\ -0.176 \\ -0.026 \\ -0979 \\ -0.118 \\ 0.033 \\ -0.616 \\ -0.079 \\ 0.009 \\ -0.485 \\ -0.070 \\ 0.003 \\ -0.485 \\ -0.070 \\ 0.003 \\ -0.048 \\ -0.003 \\ -0.033 \\ -0.003 \\ -0.263 \\ -0.033 \\ -0.005 \\ -0.201 \\ \alpha_0 = 1.5, \beta_0 = \\ \hline \text{Relative bias} \\ -0.162 \\ 0.005 \\ -0.828 \\ -0.099 \\ 0.024 \\ -0.629 \\ -0.061 \\ 0.009 \\ -0.358 \\ -0.055 \\ 0.021 \\ -0.389 \\ -0.043 \\ 0.012 \\ -0.278 \\ -0.036 \\ 0.009 \end{array}$	$\begin{array}{r} 2, \gamma_0 = 5, p\\ \hline \text{variance}\\ 000.428\\ 003.262\\ 141.060\\ 000.267\\ 001.414\\ 071.433\\ 000.195\\ 000.903\\ 053.834\\ 000.167\\ 000.691\\ 049.919\\ 000.126\\ 000.608\\ 029.810\\ 000.0427\\ 018.784\\ 3.5, \gamma_0 = 5,\\ \text{variance}\\ 01.528\\ 02.854\\ 93.549\\ 00.451\\ 01.212\\ 53.752\\ 00.298\\ 00.872\\ 27.697\\ 00.232\\ 00.725\\ 29.232\\ 00.166\\ 00.567\\ 17.054\\ 00.142\\ 00.$	$\begin{array}{l} \mathbf{p} = 6.5 \\ \mathbf{RMSE} \\ 00.744 \\ 01.811 \\ 13.475 \\ 00.569 \\ 01.189 \\ 09.352 \\ 00.469 \\ 00.952 \\ 07.985 \\ 00.433 \\ 00.832 \\ 07.529 \\ 00.364 \\ 00.781 \\ 05.722 \\ 00.364 \\ 00.781 \\ 05.722 \\ 00.364 \\ 00.756 \\ 1.360 \\ 1.360 \\ 1.360 \\ 1.360 \\ 1.360 \\ 1.365 \\ 5.829 \\ 0.556 \\ 0.935 \\ 5.829 \\ 0.518 \\ 0.857 \\ 6.0555 \\ 0.436 \\ 0.756 \\ 0.397 \\ 6.0555 \\ 0.436 \\ 0.756 \\ 0.397 \\ 6.0555 \\ 0.436 \\ 0.756 \\ 0.397 \\ 6.055 \\ 0.436 \\ 0.756 \\ 0.397 \\ 0.577 \\ 0.597 \\ 0$	$\begin{array}{r} {\rm Scaled \ RMSE} \\ 0.372 \\ 0.362 \\ 2.073 \\ 0.284 \\ 0.238 \\ 1.439 \\ 0.235 \\ 0.190 \\ 1.229 \\ 0.216 \\ 0.166 \\ 1.158 \\ 0.182 \\ 0.156 \\ 0.182 \\ 0.156 \\ 0.182 \\ 0.155 \\ 0.131 \\ 0.696 \\ \hline \end{array}$
n=50 $n=100$ $n=150$ $n=200$ $n=300$ $n=50$ $n=100$ $n=150$ $n=200$ $n=250$ $n=200$ $n=300$	$ \begin{array}{c} \beta \\ \gamma \\ \mathbf{p} \\ \mathbf{p} \\ \beta \\ \gamma \\ \gamma \\ \mathbf{p} \\ \beta \\ \gamma \\ \gamma$	$\begin{array}{r} 3.\\ \text{Mean}\\ 02.353\\ 05.134\\ 12.865\\ 02.237\\ 04.984\\ 10.503\\ 02.158\\ 04.953\\ 09.652\\ 02.141\\ 04.987\\ 09.101\\ 02.084\\ 05.043\\ 08.212\\ 02.065\\ 05.023\\ 07.803\\ 08.212\\ 02.065\\ 05.023\\ 07.803\\ 04.972\\ 12.800\\ 03.848\\ 04.972\\ 12.800\\ 03.848\\ 04.972\\ 12.800\\ 03.848\\ 04.972\\ 03.848\\ 04.972\\ 03.625\\ 04.894\\ 09.726\\ 03.651\\ 04.941\\ 08.947\\ 03.625\\ 04.956\\ 04.956\\ 03.651\\ 04.956\\ 04.956\\ 03.655\\ 04.956\\ 03.655\\ 04.956\\ 04.956\\ 03.655\\ 04.956\\ 04.956\\ 04.956\\ 04.956\\ 04.956\\ 04.956\\ 04.956\\ 04.956\\ 04.956\\ 05.75\\ 04.956\\ 04.956\\ 04.956\\ 04.956\\ 04.956\\ 04.956\\ 04.956\\ 04.956\\ 04.956\\ 04.956\\ 04.956\\ 04.956\\ 04.956\\ 04.956\\ 04.956\\ 05.75\\ 04.956\\ 04.956\\ 05.75\\ 05.75\\ 05.7$	$\begin{array}{r} \alpha_0 = 1.2, \beta_0 = \\ \hline \text{Relative bias} \\ -0.176 \\ -0.026 \\ -0979 \\ -0.118 \\ 0.033 \\ -0.616 \\ -0.079 \\ 0.009 \\ -0.485 \\ -0.070 \\ 0.009 \\ -0.485 \\ -0.070 \\ 0.003 \\ -0.485 \\ -0.070 \\ 0.003 \\ -0.400 \\ -0.042 \\ -0.003 \\ -0.003 \\ -0.003 \\ -0.005 \\ -0.263 \\ -0.005 \\ -0.263 \\ -0.005 \\ -0.263 \\ -0.005 \\ -0.263 \\ -0.005 \\ -0.263 \\ -0.005 \\ -0.263 \\ -0.005 \\ -0.261 \\ 0.005 \\ -0.024 \\ -0.629 \\ -0.061 \\ 0.009 \\ -0.358 \\ -0.055 \\ 0.021 \\ -0.389 \\ -0.043 \\ 0.012 \\ -0.278 \\ -0.036 \\ 0.008 \\ 0.008 \\ 0.05 \end{array}$	$\begin{array}{r} 2, \gamma_0 = 5, p\\ \hline \text{variance}\\ 000.428\\ 003.262\\ 141.060\\ 000.267\\ 001.414\\ 071.433\\ 000.195\\ 000.903\\ 053.834\\ 000.195\\ 000.903\\ 053.834\\ 000.195\\ 000.903\\ 053.834\\ 000.195\\ 000.903\\ 053.834\\ 000.195\\ 000.903\\ 000.903\\ 000.912\\ 000.691\\ 000.126\\ 000.691\\ 000.126\\ 000.691\\ 000.126\\ 000.691\\ 000.126\\ 000.691\\ 000.126\\ 000.427\\ 018.784\\ 029.810\\ 000.427\\ 018.784\\ 029.810\\ 000.427\\ 018.784\\ 000.427\\ 018.784\\ 000.427\\ 018.784\\ 000.427\\ 000.232\\ 00.725\\ 29.232\\ 00.765\\ 29.232\\ 00.166\\ 00.567\\ 17.054\\ 00.142\\ 00.484\\ 15.226\end{array}$	$\begin{array}{c} \mathbf{p} = 6.5 \\ \mathbf{RMSE} \\ 00.744 \\ 01.811 \\ 13.475 \\ 00.569 \\ 01.189 \\ 09.352 \\ 00.469 \\ 00.952 \\ 07.985 \\ 00.433 \\ 00.832 \\ 07.529 \\ 00.364 \\ 00.781 \\ 05.722 \\ 00.364 \\ 00.781 \\ 05.722 \\ 00.364 \\ 0.781 \\ 05.722 \\ 00.364 \\ 0.756 \\ 1.107 \\ 8.554 \\ 0.756 \\ 1.107 \\ 8.554 \\ 0.586 \\ 0.935 \\ 5.829 \\ 0.518 \\ 0.857 \\ 6.055 \\ 0.436 \\ 0.756 \\ 4.566 \\ 0.756 \\ 4.566 \\ 0.397 \\ 0.6$	$\begin{array}{c} \text{Scaled RMSE} \\ 0.372 \\ 0.362 \\ 2.073 \\ 0.284 \\ 0.238 \\ 1.439 \\ 0.235 \\ 0.190 \\ 1.229 \\ 0.216 \\ 0.166 \\ 1.158 \\ 0.380 \\ 0.155 \\ 0.182 \\ 0.155 \\ 0.182 \\ 0.155 \\ 0.131 \\ 0.696 \\ \hline \\ \hline \\ \text{Scaled RMSE} \\ 0.388 \\ 0.338 \\ 1.611 \\ 0.216 \\ 0.222 \\ 1.222 \\ 0.167 \\ 0.833 \\ 0.148 \\ 0.388 \\ 0.338 \\ 1.611 \\ 0.216 \\ 0.222 \\ 1.222 \\ 0.167 \\ 0.833 \\ 0.148 \\ 0.172 \\ 0.865 \\ 0.151 \\ 0.652 \\ 0.113 \\ 0.139 \\ 0.139 \\ 0.532 \\ 0.139 \\ 0.512 \\ 0.512 \\ 0.139 \\ 0.512 \\ 0.512 \\ 0.139 \\ 0.512 \\ 0.512 \\ 0.139 \\ 0.512$

Table 3.	MLE's and mean	square e	error of th	iree paran	neters ($eta,\gamma{ m ano}$	dp)iı	ı the	\mathbf{case}
	of the	$(\mathbf{P}-\mathbf{A}-\mathbf{L})$	modified	Weibull d	listributi	ion			

		1. α_0	$_{0}=1.25, eta_{0}=2$	$.5, \gamma_0 = 0.5$	$, p_0 = 0.9$	
·		Mean	Relative bias	variance	RMSE	Scaled RMSE
n=50	α	1.745	-0.396	0001.321	1.252	1.001
	B	2.382	0.047	0001.271	1.133	0.453
	γ	0.488	0.024	0000.009	0.099	0.198
	\mathbf{p}	2.654	-1.949	1234.48	35.178	39.087
n = 100	α	1.545	-0.236	0000.750	0.915	0.732
	B	2.415	0.033	0000.904	0.954	0.381
	γ	0.485	0.029	0000.004	0.068	0.136
	\mathbf{p}	1.217	-0.352	0000.786	0.941	1.046
n = 150	α	1.436	-0.149	0000.712	0.864	0.691
	β	2.439	0.024	0000.959	0.981	0.392
	γ	0.491	0.016	0000.003	0.057	0.115
	p	1.179	-0.310	0000.848	0.962	1.069
n=200	α	1.304	-0.043	0004.706	2.170	1.736
	β	2.589	-0.035	0005.201	2.282	0.912
	γ	0.490	0.019	0000.004	0.065	1 102
	$\frac{p}{q}$	1.184	-0.315	0000.903	0.992	0.564
n=250	B	2 478	-0.075	0000.489	0.700	0.304
	P	0.490	0.008	0000.747	0.805	0.340
	n	1.114	-0.237	0000.001	0.045	0.829
n=300	$\frac{\rho}{\alpha}$	1.317	-0.053	0000.011	0.696	0.556
n=300	B	2.495	-0.001	0000.731	0.855	0.342
	\sim	0.494	0.010	0000.002	0.043	0.086
	p	1.089	-0.210	0000.462	0.706	0.784
		2. 0	$\alpha = 0.2, \beta_0 = 2$	$2. \gamma_0 = 2.8$	$p_0 = 0.8$	
L		Mean	Belative bias	variance	BMSE	Scaled BMSE
n-50	0	0.380	0.948	0.251	0.536	2.679
n=50	B	2.304	-0.047	0.251 0.352	0.602	0.274
	~	2.958	-0.056	0.417	0.665	0.237
	'n	1.408	-0.760	7.616	2.679	3.533
n=100	$\frac{P}{\alpha}$	0.263	-0.316	0.053	0.238	1.193
	B	2.284	-0.038	0.139	0.382	0.174
	γ	2.903	-0.037	0.285	0.543	0.194
	p	0.958	-0.197	0.359	0.533	0.666
		Moan	Bolativo bias	varianco	BMSE	Scaled BMSE
		TATOOTT		variance	TUTUTOTO	beated remains
n-150	0	0 999	0.162	0.018	0.120	0.695
n=150	$\frac{\alpha}{\beta}$	0.232 2.257	-0.163	0.018 0.073	$0.139 \\ 0.276$	0.695 0.125
n=150	$\alpha \\ \beta \\ \gamma$	$0.232 \\ 2.257 \\ 2.831$	-0.163 0.026 -0.011	$0.018 \\ 0.073 \\ 0.094$	$0.139 \\ 0.276 \\ 0.308$	$0.695 \\ 0.125 \\ 0.110$
n=150	$egin{array}{c} lpha \ eta \ \gamma \ p \end{array}$	$0.232 \\ 2.257 \\ 2.831 \\ 0.940$	-0.163 0.026 -0.011 -0.175	$\begin{array}{c} 0.018 \\ 0.073 \\ 0.094 \\ 0.152 \end{array}$	$0.139 \\ 0.276 \\ 0.308 \\ 0.414$	$0.695 \\ 0.125 \\ 0.110 \\ 0.517$
n=150 n=200	$egin{array}{c} lpha \ eta \ \gamma \ p \ lpha \ \lpha \ lpha \ \lpha \ \ \lpha \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \$	$\begin{array}{c} 0.232 \\ 2.257 \\ 2.831 \\ 0.940 \\ 0.219 \end{array}$	$-0.163 \\ 0.026 \\ -0.011 \\ -0.175 \\ -0.098$	$\begin{array}{c} 0.018 \\ 0.073 \\ 0.094 \\ 0.152 \\ 0.013 \end{array}$	$\begin{array}{c} 0.139 \\ 0.276 \\ 0.308 \\ 0.414 \\ 0.116 \end{array}$	$\begin{array}{c} 0.695 \\ 0.125 \\ 0.110 \\ 0.517 \\ 0.584 \end{array}$
n=150 n=200	$\begin{array}{c} \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \end{array}$	$\begin{array}{c} 0.232 \\ 2.257 \\ 2.831 \\ 0.940 \\ 0.219 \\ 2.255 \end{array}$	$\begin{array}{r} -0.163\\ 0.026\\ -0.011\\ -0.175\\ -0.098\\ -0.025\end{array}$	$\begin{array}{c} 0.018 \\ 0.073 \\ 0.094 \\ 0.152 \\ 0.013 \\ 0.054 \end{array}$	$\begin{array}{c} 0.139 \\ 0.276 \\ 0.308 \\ 0.414 \\ 0.116 \\ 0.0239 \end{array}$	$\begin{array}{c} 0.695 \\ 0.125 \\ 0.110 \\ 0.517 \\ 0.584 \\ 0.108 \end{array}$
n=150 n=200	$\begin{array}{c} \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \end{array}$	$\begin{array}{c} 0.232 \\ 2.257 \\ 2.831 \\ 0.940 \\ 0.219 \\ 2.255 \\ 2.808 \\ 0.98 \end{array}$	$\begin{array}{c} -0.163\\ 0.026\\ -0.011\\ -0.175\\ -0.098\\ -0.025\\ -0.002\\ -0.002\\ \end{array}$	$\begin{array}{c} 0.018 \\ 0.073 \\ 0.094 \\ 0.152 \\ 0.013 \\ 0.054 \\ 0.073 \\ 0.120 \end{array}$	$\begin{array}{c} 0.139\\ 0.276\\ 0.308\\ 0.414\\ 0.116\\ 0.0239\\ 0.271\\ 0.271\\ 0.972\end{array}$	$\begin{array}{c} 0.695\\ 0.125\\ 0.110\\ 0.517\\ 0.584\\ 0.108\\ 0.096\\ 0.422\\ \end{array}$
n=150 n=200	$\begin{array}{c} \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ p \\ p \end{array}$	$\begin{array}{c} 0.232 \\ 2.257 \\ 2.831 \\ 0.940 \\ 0.219 \\ 2.255 \\ 2.808 \\ 0.930 \\ 0.931 \end{array}$	$\begin{array}{r} -0.163\\ 0.026\\ -0.011\\ -0.175\\ -0.098\\ -0.025\\ -0.002\\ -0.163\\ -0.25\end{array}$	$\begin{array}{c} 0.018\\ 0.073\\ 0.094\\ 0.152\\ 0.013\\ 0.054\\ 0.073\\ 0.120\\ 0.010\end{array}$	$\begin{array}{c} 0.139\\ 0.276\\ 0.308\\ 0.414\\ 0.116\\ 0.0239\\ 0.271\\ 0.370\\ 0.101\end{array}$	$\begin{array}{c} 0.695\\ 0.125\\ 0.110\\ 0.517\\ 0.584\\ 0.108\\ 0.096\\ 0.463\\ 0.567\\ \end{array}$
n=150 n=200 n=250	$\begin{array}{c} \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \beta$	$\begin{array}{c} 0.232\\ 2.257\\ 2.831\\ 0.940\\ 0.219\\ 2.255\\ 2.808\\ 0.930\\ 0.215\\ 2.274 \end{array}$	$\begin{array}{r} -0.163\\ 0.026\\ -0.011\\ -0.175\\ -0.098\\ -0.025\\ -0.002\\ -0.163\\ -0.079\\ -0.079\\ -0.033\end{array}$	$\begin{array}{c} 0.018\\ 0.073\\ 0.094\\ 0.152\\ 0.013\\ 0.054\\ 0.073\\ 0.120\\ 0.010\\ 0.051\\ \end{array}$	$\begin{array}{c} 0.139\\ 0.276\\ 0.308\\ 0.414\\ 0.116\\ 0.0239\\ 0.271\\ 0.370\\ 0.101\\ 0.237\end{array}$	$\begin{array}{c} 0.695\\ 0.125\\ 0.110\\ 0.517\\ 0.584\\ 0.108\\ 0.096\\ 0.463\\ 0.507\\ 0.108\\ \end{array}$
n=150 n=200 n=250	$\begin{array}{c} \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \end{array}$	$\begin{array}{c} 0.232\\ 2.257\\ 2.831\\ 0.940\\ 0.219\\ 2.255\\ 2.808\\ 0.930\\ 0.215\\ 2.274\\ 2.789\end{array}$	$\begin{array}{c} -0.163\\ 0.026\\ -0.011\\ -0.175\\ -0.098\\ -0.025\\ -0.002\\ -0.163\\ -0.079\\ -0.033\\ 0.003\end{array}$	$\begin{array}{c} 0.018\\ 0.073\\ 0.094\\ 0.152\\ 0.013\\ 0.054\\ 0.073\\ 0.120\\ 0.010\\ 0.051\\ 0.055\end{array}$	$\begin{array}{c} 0.139\\ 0.276\\ 0.308\\ 0.414\\ 0.116\\ 0.0239\\ 0.271\\ 0.370\\ 0.101\\ 0.237\\ 0.236\\ \end{array}$	$\begin{array}{c} 0.695\\ 0.125\\ 0.110\\ 0.517\\ 0.584\\ 0.108\\ 0.096\\ 0.463\\ 0.507\\ 0.108\\ 0.084\\ \end{array}$
n=150 n=200 n=250	$\begin{array}{c} \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ p \end{array}$	$\begin{array}{c} 0.232\\ 2.257\\ 2.831\\ 0.940\\ 0.219\\ 2.255\\ 2.808\\ 0.930\\ 0.215\\ 2.274\\ 2.789\\ 0.951\\ \end{array}$	$\begin{array}{c} -0.163\\ 0.026\\ -0.011\\ -0.175\\ -0.098\\ -0.025\\ -0.002\\ -0.163\\ -0.079\\ -0.033\\ 0.003\\ -0.189\end{array}$	$\begin{array}{c} 0.018\\ 0.073\\ 0.094\\ 0.152\\ 0.013\\ 0.054\\ 0.073\\ 0.120\\ 0.010\\ 0.051\\ 0.055\\ 0.143\\ \end{array}$	$\begin{array}{c} 0.139\\ 0.276\\ 0.308\\ 0.414\\ 0.116\\ 0.0239\\ 0.271\\ 0.370\\ 0.101\\ 0.237\\ 0.236\\ 0.407 \end{array}$	$\begin{array}{c} 0.695\\ 0.125\\ 0.110\\ 0.517\\ 0.584\\ 0.108\\ 0.096\\ 0.463\\ 0.507\\ 0.108\\ 0.084\\ 0.084\\ 0.509\end{array}$
n=150 n=200 n=250	$ \begin{array}{c} \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ p \end{array} $	$\begin{array}{c} 0.232\\ 2.257\\ 2.831\\ 0.940\\ 0.219\\ 2.255\\ 2.808\\ 0.930\\ 0.215\\ 2.274\\ 2.789\\ 0.951\\ 3.\end{array}$	$\begin{array}{c} -0.163\\ 0.026\\ -0.011\\ -0.175\\ -0.098\\ -0.025\\ -0.002\\ -0.163\\ -0.079\\ -0.033\\ 0.003\\ -0.189\\ \hline \alpha_0 = 1.2, \beta_0 = \end{array}$	$\begin{array}{c} 0.018\\ 0.073\\ 0.094\\ 0.152\\ 0.013\\ 0.054\\ 0.073\\ 0.120\\ 0.010\\ 0.051\\ 0.055\\ 0.143\\ 2, \gamma_0=5, p_{\rm c}\end{array}$	$\begin{array}{c} 0.139\\ 0.276\\ 0.308\\ 0.414\\ 0.116\\ 0.0239\\ 0.271\\ 0.370\\ 0.101\\ 0.237\\ 0.236\\ 0.407\\ 0=6.5 \end{array}$	$\begin{array}{c} 0.695\\ 0.125\\ 0.110\\ 0.517\\ 0.584\\ 0.108\\ 0.096\\ 0.463\\ 0.507\\ 0.108\\ 0.084\\ 0.084\\ 0.509\\ \end{array}$
n=150 n=200 n=250	$\begin{array}{c} \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ p \end{array}$	0.232 2.257 2.831 0.940 0.219 2.255 2.808 0.930 0.215 2.274 2.789 0.951 3. Mean	$\begin{array}{c} -0.163\\ 0.026\\ -0.011\\ -0.175\\ -0.098\\ -0.025\\ -0.002\\ -0.163\\ -0.079\\ -0.033\\ 0.003\\ -0.189\\ \hline \alpha_0 = 1.2, \beta_0 = \\ \text{Relative bias} \end{array}$	$\begin{array}{c} 0.018\\ 0.073\\ 0.094\\ 0.152\\ 0.013\\ 0.054\\ 0.073\\ 0.120\\ 0.010\\ 0.051\\ 0.055\\ 0.143\\ 2, \gamma_0=5, p_0\\ \mathrm{variance} \end{array}$	$\begin{array}{c} 0.139\\ 0.276\\ 0.308\\ 0.414\\ 0.116\\ 0.0239\\ 0.271\\ 0.370\\ 0.101\\ 0.237\\ 0.236\\ 0.407\\ 0.= 6.5\\ \textbf{RMSE} \end{array}$	0.695 0.125 0.110 0.517 0.584 0.108 0.096 0.463 0.507 0.108 0.507 0.108 0.084 0.509 Scaled RMSE
n=150 n=200 n=250 	$\begin{array}{c} \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ p \\ \end{array}$	$\begin{array}{c} 0.232\\ 2.257\\ 2.831\\ 0.940\\ 0.219\\ 2.255\\ 2.808\\ 0.930\\ 0.215\\ 2.274\\ 2.789\\ 0.951\\ \hline 3.\\ \\ Mean\\ 1.286\end{array}$	$\begin{array}{c} -0.163\\ 0.026\\ -0.011\\ -0.175\\ -0.098\\ -0.025\\ -0.002\\ -0.163\\ -0.079\\ -0.033\\ 0.003\\ -0.189\\ \hline \alpha_0=1.2, \beta_0=\\ \hline \text{Relative bias}\\ -0.072 \end{array}$	$\begin{array}{c} 0.018\\ 0.073\\ 0.094\\ 0.152\\ 0.013\\ 0.054\\ 0.073\\ 0.120\\ 0.010\\ 0.055\\ 0.143\\ 2, \gamma_0 = 5, p_{\rm variance}\\ 0.0313 \end{array}$	$\begin{array}{c} 0.139\\ 0.276\\ 0.308\\ 0.414\\ 0.116\\ 0.0239\\ 0.271\\ 0.370\\ 0.101\\ 0.237\\ 0.236\\ 0.407\\ 0.= 6.5\\ \hline \text{RMSE}\\ 0.566 \end{array}$	$\begin{array}{c} 0.695\\ 0.125\\ 0.110\\ 0.517\\ 0.584\\ 0.108\\ 0.096\\ 0.463\\ 0.507\\ 0.108\\ 0.084\\ 0.509\\ \hline \end{array}$
n=150 n=200 n=250 n=50	$\begin{array}{c} \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ p \\ \end{array}$	$\begin{array}{c} 0.232\\ 2.257\\ 2.831\\ 0.940\\ 0.219\\ 2.255\\ 2.808\\ 0.930\\ 0.215\\ 2.274\\ 2.789\\ 0.951\\ \hline & & \\ 3.\\ \hline & \\ Mean\\ 1.286\\ 2.078 \end{array}$	$\begin{array}{c} -0.163\\ 0.026\\ -0.011\\ -0.175\\ -0.098\\ -0.025\\ -0.002\\ -0.163\\ -0.079\\ -0.033\\ 0.003\\ -0.079\\ -0.038\\ 0.003\\ -0.089\\ \hline \alpha_0 = 1.2, \beta_0 = \\ \hline \text{Relative bias}\\ -0.072\\ -0.039\\ \end{array}$	$\begin{array}{c} 0.018\\ 0.073\\ 0.094\\ 0.152\\ 0.013\\ 0.054\\ 0.073\\ 0.120\\ 0.010\\ 0.051\\ 0.055\\ 0.143\\ 2, \gamma_0=5, p_0\\ \text{variance}\\ 00.313\\ 00.324 \end{array}$	$\begin{array}{c} 0.139\\ 0.276\\ 0.308\\ 0.414\\ 0.116\\ 0.0239\\ 0.271\\ 0.370\\ 0.101\\ 0.237\\ 0.236\\ 0.407\\ 0 = 6.5\\ \hline \mathbf{RMSE}\\ 0.566\\ 0.574 \end{array}$	$\begin{array}{c} 0.695\\ 0.125\\ 0.110\\ 0.517\\ 0.584\\ 0.108\\ 0.096\\ 0.463\\ 0.507\\ 0.108\\ 0.084\\ 0.509\\ \hline \end{array}$
n=150 n=200 n=250 n=50	$\begin{array}{c} \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ p \end{array}$	$\begin{array}{c} 0.232\\ 2.257\\ 2.831\\ 0.940\\ 0.219\\ 2.255\\ 2.808\\ 0.930\\ 0.215\\ 2.274\\ 2.789\\ 0.951\\ 3.\\ Mean\\ 1.286\\ 2.078\\ 5.710\\ \end{array}$	$\begin{array}{c} -0.163\\ 0.026\\ -0.011\\ -0.175\\ -0.098\\ -0.025\\ -0.002\\ -0.163\\ -0.079\\ -0.033\\ 0.003\\ -0.189\\ \hline \alpha_0 = 1.2, \beta_0 =\\ \hline \text{Relative bias}\\ -0.072\\ -0.039\\ -0.142\\ \end{array}$	$\begin{array}{c} 0.018\\ 0.073\\ 0.094\\ 0.152\\ 0.013\\ 0.054\\ 0.073\\ 0.120\\ 0.010\\ 0.051\\ 0.055\\ 0.143\\ 2, \gamma_0 = 5, p_0\\ \hline \text{variance}\\ 00.313\\ 00.324\\ 04.127\\ \end{array}$	$\begin{array}{c} 0.139\\ 0.276\\ 0.308\\ 0.414\\ 0.116\\ 0.0239\\ 0.271\\ 0.370\\ 0.101\\ 0.237\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.566\\ 0.556\\ 0.5564\\ 2.152 \end{array}$	$\begin{array}{c} 0.695\\ 0.125\\ 0.110\\ 0.517\\ 0.584\\ 0.108\\ 0.096\\ 0.463\\ 0.507\\ 0.108\\ 0.084\\ 0.509\\ \hline \end{array}$
n=150 n=200 n=250 n=50	$\begin{array}{c} \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ p \end{array}$	$\begin{array}{c} 0.232\\ 2.257\\ 2.831\\ 0.940\\ 0.219\\ 2.255\\ 2.808\\ 0.930\\ 0.215\\ 2.274\\ 2.789\\ 0.951\\ \hline & 3.\\ \hline & \\ 3.\\ \hline & \\ Mean\\ 1.286\\ 2.078\\ 5.710\\ 8.412\\ \hline & \\ 5.710\\ 8.412\\ \hline & \\ 5.551\\ \hline & \\$	$\begin{array}{c} -0.163\\ 0.026\\ -0.011\\ -0.175\\ -0.098\\ -0.025\\ -0.002\\ -0.163\\ -0.079\\ -0.033\\ 0.003\\ -0.189\\ \hline \alpha_0=1.2, \beta_0=\\ \hline \text{Relative bias}\\ -0.072\\ -0.039\\ -0.142\\ -0.294\\ -0.294\\ -0.555\\ \hline \end{array}$	$\begin{array}{c} 0.018\\ 0.073\\ 0.094\\ 0.152\\ 0.013\\ 0.054\\ 0.073\\ 0.120\\ 0.010\\ 0.055\\ 0.143\\ 2, \gamma_0=5, p_1\\ \text{variance}\\ 00.313\\ 00.324\\ 04.127\\ 30.485\\ \end{array}$	$\begin{array}{c} 0.139\\ 0.276\\ 0.308\\ 0.414\\ 0.116\\ 0.0239\\ 0.271\\ 0.370\\ 0.101\\ 0.237\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ \textbf{RMSE}\\ 0.566\\ 0.574\\ 2.152\\ 5.843\\ 0.564\\ 0.574\\ 0.5843\\ 0.566\\ 0.574\\ 0.5843\\ 0.5844\\ 0.5844\\ 0.5$	$\begin{array}{c} 0.695\\ 0.125\\ 0.110\\ 0.517\\ 0.584\\ 0.108\\ 0.096\\ 0.463\\ 0.507\\ 0.108\\ 0.507\\ 0.108\\ 0.509\\ \hline \end{array}$
n=150 n=200 n=250 n=50 n=100	$\begin{array}{c} \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ \gamma$	$\begin{array}{c} 0.232\\ 2.257\\ 2.831\\ 0.940\\ 0.219\\ 2.255\\ 2.808\\ 0.930\\ 0.215\\ 2.274\\ 2.789\\ 0.951\\ \hline \end{array}$	$\begin{array}{c} -0.163\\ 0.026\\ -0.011\\ -0.175\\ -0.098\\ -0.025\\ -0.002\\ -0.163\\ -0.079\\ -0.033\\ 0.003\\ -0.189\\ \hline \alpha_0 = 1.2, \beta_0 =\\ \hline \text{Relative bias}\\ -0.072\\ -0.039\\ -0.142\\ -0.0294\\ -0.018\\ -0.0294\\ \hline \end{array}$	$\begin{array}{c} 0.018\\ 0.073\\ 0.094\\ 0.152\\ 0.013\\ 0.054\\ 0.073\\ 0.120\\ 0.010\\ 0.051\\ 0.055\\ 0.143\\ 2, \gamma_0 = 5, p_1\\ \text{variance}\\ 00.313\\ 00.324\\ 04.127\\ 30.485\\ 00.131\\ 00.152\\ \end{array}$	$\begin{array}{c} 0.139\\ 0.276\\ 0.308\\ 0.414\\ 0.116\\ 0.0239\\ 0.271\\ 0.370\\ 0.101\\ 0.237\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.576\\ RMSE\\ 0.566\\ 0.574\\ 2.152\\ 5.843\\ 0.362\\ 0.362\\ 0.362\\ \end{array}$	$\begin{array}{c} 0.695\\ 0.125\\ 0.110\\ 0.517\\ 0.584\\ 0.096\\ 0.463\\ 0.507\\ 0.108\\ 0.084\\ 0.507\\ 0.108\\ 0.084\\ 0.509\\ \hline \end{array}$
n=150 n=200 n=250 n=50 n=100	$\begin{array}{c} \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ \beta \\ \gamma \\ \beta \\ \gamma \\ \gamma \\ \gamma \\ \gamma \\ \gamma$	$\begin{array}{c} 0.232\\ 2.257\\ 2.831\\ 0.940\\ 0.219\\ 2.255\\ 2.808\\ 0.930\\ 0.215\\ 2.274\\ 2.789\\ 0.951\\ \hline \\ 3.\\ \hline \\ Mean\\ 1.286\\ 2.078\\ 5.710\\ 8.412\\ 1.222\\ 2.058\\ 5.262\\ \hline \end{array}$	$\begin{array}{c} -0.163\\ 0.026\\ -0.011\\ -0.175\\ -0.098\\ -0.025\\ -0.002\\ -0.163\\ -0.079\\ -0.033\\ 0.003\\ -0.079\\ -0.038\\ 0.003\\ -0.189\\ \hline \alpha_0 = 1.2, \beta_0 = \\ \hline \text{Relative bias}\\ -0.072\\ -0.039\\ -0.142\\ -0.294\\ -0.018\\ -0.028\\ -$	$\begin{array}{c} 0.018\\ 0.073\\ 0.094\\ 0.152\\ 0.013\\ 0.054\\ 0.073\\ 0.120\\ 0.010\\ 0.051\\ 0.055\\ 0.143\\ 2, \gamma_0 = 5, p_0\\ \hline \text{variance}\\ 00.313\\ 00.324\\ 04.127\\ 30.485\\ 00.131\\ 00.152\\ 01.1227\\ \end{array}$	$\begin{array}{c} 0.139\\ 0.276\\ 0.308\\ 0.414\\ 0.116\\ 0.0239\\ 0.271\\ 0.370\\ 0.101\\ 0.237\\ 0.236\\ 0.407\\ 0 = 6.5\\ \hline \textbf{RMSE}\\ \hline \textbf{RMSE}\\ 0.566\\ 0.574\\ 2.152\\ 5.843\\ 0.362\\ 0.394\\ 1.142\\ \end{array}$	$\begin{array}{c} 0.695\\ 0.125\\ 0.110\\ 0.517\\ 0.584\\ 0.096\\ 0.463\\ 0.507\\ 0.108\\ 0.084\\ 0.509\\ \hline \end{array}$
n=150 n=200 n=250 n=50 n=100	$\begin{array}{c} \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ \gamma$	$\begin{array}{c} 0.232\\ 2.257\\ 2.831\\ 0.940\\ 0.219\\ 2.255\\ 2.808\\ 0.930\\ 0.215\\ 2.274\\ 2.789\\ 0.951\\ \hline \\ 3.\\ Mean\\ 1.286\\ 2.078\\ 5.710\\ 8.412\\ 1.222\\ 2.058\\ 5.263\\ 5.263\\ 7.522 \end{array}$	$\begin{array}{c} -0.163\\ 0.026\\ -0.011\\ -0.175\\ -0.098\\ -0.025\\ -0.002\\ -0.163\\ -0.079\\ -0.033\\ 0.003\\ -0.189\\ \hline \alpha_0=1.2, \beta_0=\\ \hline \text{Relative bias}\\ -0.072\\ -0.039\\ -0.142\\ -0.294\\ -0.018\\ -0.028\\ -0.053\\ -0.053\\ -0.157\\ \end{array}$	$\begin{array}{c} 0.018\\ 0.073\\ 0.094\\ 0.152\\ 0.013\\ 0.054\\ 0.073\\ 0.120\\ 0.010\\ 0.051\\ 0.055\\ 0.143\\ 2, \gamma_0=5, p_0\\ \text{variance}\\ 00.313\\ 00.324\\ 04.127\\ 30.485\\ 00.131\\ 00.152\\ 01.237\\ 16.419 \end{array}$	$\begin{array}{c} 0.139\\ 0.276\\ 0.308\\ 0.414\\ 0.116\\ 0.0239\\ 0.271\\ 0.370\\ 0.101\\ 0.237\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.566\\ 0.574\\ 2.152\\ 5.843\\ 0.362\\ 0.394\\ 1.143\\ 4.179\\ \end{array}$	$\begin{array}{c} 0.695\\ 0.125\\ 0.110\\ 0.517\\ 0.584\\ 0.108\\ 0.096\\ 0.463\\ 0.507\\ 0.108\\ 0.507\\ 0.108\\ 0.509\\ \hline \end{array}$
n=150 n=200 n=250 n=50 n=100 n=150	$\begin{array}{c} \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ \beta \\ \alpha \\ \alpha$	$\begin{array}{c} 0.232\\ 2.257\\ 2.831\\ 0.940\\ 0.219\\ 2.255\\ 2.808\\ 0.930\\ 0.215\\ 2.274\\ 2.789\\ 0.951\\ \hline 3.\\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$	$\begin{array}{c} -0.163\\ 0.026\\ -0.011\\ -0.175\\ -0.098\\ -0.025\\ -0.002\\ -0.163\\ -0.079\\ -0.033\\ 0.003\\ -0.189\\ \hline \alpha_0 = 1.2, \beta_0 =\\ \hline \text{Relative bias}\\ \hline \alpha_0 = 1.2, \beta_0 =\\ \hline \text{Relative bias}\\ -0.072\\ -0.039\\ -0.142\\ -0.294\\ -0.018\\ -0.028\\ -0.053\\ -0.157\\ 0.004\\ \end{array}$	$\begin{array}{c} 0.018\\ 0.073\\ 0.094\\ 0.152\\ 0.013\\ 0.054\\ 0.073\\ 0.120\\ 0.010\\ 0.055\\ 0.143\\ 2, \gamma_0 = 5, p_{\rm v}\\ {\rm variance}\\ 00.313\\ 00.324\\ 04.127\\ 30.485\\ 00.131\\ 00.152\\ 01.237\\ 16.419\\ 00.076\end{array}$	$\begin{array}{c} 0.139\\ 0.276\\ 0.308\\ 0.414\\ 0.116\\ 0.0239\\ 0.271\\ 0.370\\ 0.101\\ 0.237\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.574\\ 2.152\\ 5.843\\ 0.362\\ 0.394\\ 1.143\\ 4.179\\ 0.275\\ \end{array}$	$\begin{array}{c} 0.695\\ 0.125\\ 0.110\\ 0.517\\ 0.584\\ 0.108\\ 0.096\\ 0.463\\ 0.507\\ 0.108\\ 0.507\\ 0.108\\ 0.509\\ \hline \end{array}$
n=150 n=200 n=250 n=50 n=100 n=150	$\begin{array}{c} \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ \gamma$	$\begin{array}{c} 0.232\\ 2.257\\ 2.831\\ 0.940\\ 0.219\\ 2.255\\ 2.808\\ 0.930\\ 0.215\\ 2.274\\ 2.789\\ 0.951\\ \hline \\ 3.\\ \hline \\ Mean\\ 1.286\\ 2.078\\ 5.710\\ 8.412\\ \hline \\ 1.222\\ 2.058\\ 5.710\\ 8.412\\ \hline \\ 1.222\\ 2.058\\ 5.263\\ 7.522\\ \hline \\ 1.195\\ 2.049\\ \end{array}$	$\begin{array}{c} -0.163\\ 0.026\\ -0.011\\ -0.175\\ -0.098\\ -0.025\\ -0.002\\ -0.163\\ 0.003\\ -0.079\\ -0.033\\ 0.003\\ -0.189\\ \hline \alpha_0 = 1.2, \beta_0 =\\ \hline \text{Relative bias}\\ -0.072\\ -0.039\\ -0.142\\ -0.039\\ -0.142\\ -0.294\\ -0.018\\ -0.028\\ -0.053\\ -0.157\\ 0.004\\ -0.025\\ \end{array}$	$\begin{array}{c} 0.018\\ 0.073\\ 0.094\\ 0.152\\ 0.013\\ 0.054\\ 0.073\\ 0.120\\ 0.010\\ 0.051\\ 0.055\\ 0.143\\ 2, \gamma_0 = 5, p\\ \text{variance}\\ 0.313\\ 00.324\\ 04.127\\ 30.485\\ 00.131\\ 00.152\\ 01.237\\ 16.419\\ 00.076\\ 00.107\\ \end{array}$	$\begin{array}{c} 0.139\\ 0.276\\ 0.308\\ 0.414\\ 0.116\\ 0.0239\\ 0.271\\ 0.370\\ 0.237\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.574\\ 2.152\\ 0.566\\ 0.574\\ 2.152\\ 5.843\\ 0.362\\ 0.394\\ 1.143\\ 4.179\\ 0.275\\ 0.330\\ \end{array}$	$\begin{array}{c} 0.695\\ 0.125\\ 0.110\\ 0.517\\ 0.584\\ 0.108\\ 0.096\\ 0.463\\ 0.507\\ 0.108\\ 0.507\\ 0.108\\ 0.509\\ \hline \end{array}$
n=150 n=200 n=250 n=50 n=100 n=150	$\begin{array}{c} \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ \gamma \\ \beta \\ \gamma \\ \gamma \\ \beta \\ \gamma \\ \gamma \\ \gamma$	$\begin{array}{c} 0.232\\ 2.257\\ 2.831\\ 0.940\\ 0.219\\ 2.255\\ 2.808\\ 0.930\\ 0.215\\ 2.274\\ 2.789\\ 0.951\\ \hline \\ 3.\\ \hline \\ 1.286\\ 2.078\\ 5.710\\ 8.412\\ 1.222\\ 2.058\\ 5.710\\ 8.412\\ 1.222\\ 1.222\\ 2.058\\ 5.263\\ 7.522\\ \hline 1.195\\ 2.049\\ 5.150\\ \hline \end{array}$	$\begin{array}{c} -0.163\\ 0.026\\ -0.011\\ -0.175\\ -0.098\\ -0.025\\ -0.002\\ -0.163\\ -0.079\\ -0.033\\ 0.003\\ -0.189\\ \hline \alpha_0 = 1.2, \beta_0 = \\ \hline \text{Relative bias}\\ -0.072\\ -0.039\\ -0.142\\ -0.294\\ -0.018\\ -0.028\\ -0.053\\ -0.053\\ -0.157\\ \hline 0.004\\ -0.025\\ -0.030\\ \hline \end{array}$	$\begin{array}{c} 0.018\\ 0.073\\ 0.094\\ 0.152\\ 0.013\\ 0.054\\ 0.073\\ 0.120\\ 0.010\\ 0.051\\ 0.055\\ 0.143\\ 2, \gamma_0=5, p_0\\ \text{variance}\\ 00.313\\ 00.324\\ 04.127\\ 30.485\\ 00.131\\ 00.152\\ 01.237\\ 16.419\\ 00.076\\ 00.706\\ \end{array}$	$\begin{array}{c} 0.139\\ 0.276\\ 0.308\\ 0.414\\ 0.116\\ 0.0239\\ 0.271\\ 0.370\\ 0.101\\ 0.237\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.574\\ 2.152\\ 5.843\\ 0.362\\ 0.394\\ 1.143\\ 4.179\\ 0.275\\ 0.330\\ 0.854 \end{array}$	$\begin{array}{c} 0.695\\ 0.125\\ 0.110\\ 0.517\\ 0.584\\ 0.108\\ 0.096\\ 0.463\\ 0.507\\ 0.108\\ 0.507\\ 0.108\\ 0.509\\ \hline \end{array}$
n=150 n=200 n=250 n=50 n=100 n=150	$\begin{array}{c} \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ \beta \\ \alpha \\ \alpha$	$\begin{array}{c} 0.232\\ 2.257\\ 2.831\\ 0.940\\ 0.219\\ 2.255\\ 2.808\\ 0.930\\ 0.215\\ 2.274\\ 2.789\\ 0.951\\ \hline 3.\\ \hline Mean\\ 1.286\\ 2.078\\ 5.710\\ 8.412\\ 1.222\\ 2.058\\ 5.710\\ 8.412\\ 1.222\\ 1.195\\ 2.049\\ 5.150\\ 7.178\\ \end{array}$	$\begin{array}{c} -0.163\\ 0.026\\ -0.011\\ -0.175\\ -0.098\\ -0.025\\ -0.002\\ -0.163\\ -0.079\\ -0.033\\ 0.003\\ -0.189\\ \hline \alpha_0 = 1.2, \beta_0 =\\ \hline \text{Relative bias}\\ \hline \alpha_0 = 1.2, \beta_0 =\\ \hline \text{Relative bias}\\ -0.072\\ -0.039\\ -0.142\\ -0.294\\ -0.018\\ -0.028\\ -0.028\\ -0.053\\ -0.157\\ \hline 0.004\\ -0.025\\ -0.030\\ -0.104\\ \hline \end{array}$	$\begin{array}{c} 0.018\\ 0.073\\ 0.094\\ 0.152\\ 0.013\\ 0.054\\ 0.073\\ 0.120\\ 0.010\\ 0.055\\ 0.143\\ 2, \gamma_0 = 5, p_1\\ \hline variance\\ 00.313\\ 00.324\\ 04.127\\ 30.485\\ 00.131\\ 00.152\\ 01.237\\ 16.419\\ 00.076\\ 00.706\\ 12.110\\ \end{array}$	$\begin{array}{c} 0.139\\ 0.276\\ 0.308\\ 0.414\\ 0.116\\ 0.0239\\ 0.271\\ 0.370\\ 0.101\\ 0.237\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.364\\ 0.574\\ 1.143\\ 4.179\\ 0.275\\ 0.330\\ 0.854\\ 3.546\\ \end{array}$	$\begin{array}{c} 0.695\\ 0.125\\ 0.110\\ 0.517\\ 0.584\\ 0.108\\ 0.096\\ 0.463\\ 0.507\\ 0.108\\ 0.507\\ 0.108\\ 0.509\\ \hline \end{array}$
n=150 n=200 n=250 n=50 n=100 n=150 n=200	$\begin{array}{c} \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \alpha$	$\begin{array}{c} 0.232\\ 2.257\\ 2.831\\ 0.940\\ 0.219\\ 2.255\\ 2.808\\ 0.930\\ 0.215\\ 2.274\\ 2.789\\ 0.951\\ \hline 3.\\ \hline Mean\\ 1.286\\ 2.078\\ 5.710\\ 8.412\\ 1.222\\ 2.058\\ 5.710\\ 8.412\\ 1.222\\ 2.058\\ 5.263\\ 7.522\\ 1.195\\ 2.049\\ 5.150\\ 7.178\\ 1.195\\ 2.049\\ 5.150\\ 7.178\\ \hline 1.195\\ 2.049\\ 5.150\\ 7.178\\ 1.195\\ $	$\begin{array}{c} -0.163\\ 0.026\\ -0.011\\ -0.175\\ -0.098\\ -0.025\\ -0.002\\ -0.163\\ -0.079\\ -0.033\\ 0.003\\ -0.189\\ \hline \alpha_0 = 1.2, \beta_0 =\\ \hline \text{Relative bias}\\ -0.072\\ -0.039\\ -0.142\\ -0.072\\ -0.039\\ -0.142\\ -0.028\\ -0.028\\ -0.028\\ -0.028\\ -0.053\\ -0.157\\ \hline 0.004\\ -0.025\\ -0.030\\ -0.104\\ \hline 00.008\\ -0.008$	$\begin{array}{c} 0.018\\ 0.073\\ 0.094\\ 0.152\\ 0.013\\ 0.054\\ 0.073\\ 0.120\\ 0.010\\ 0.051\\ 0.055\\ 0.143\\ \hline 2, \gamma_0 = 5, p\\ \hline variance\\ 00.313\\ 00.324\\ 04.127\\ 10.485\\ 00.131\\ 00.152\\ 01.237\\ 16.419\\ 00.107\\ 00.706\\ 00.107\\ 00.706\\ 00.107\\ 00.706\\ 00.107\\ 0.052\\ \hline 0.052\\ 00.52$	$\begin{array}{c} 0.139\\ 0.276\\ 0.308\\ 0.414\\ 0.116\\ 0.0239\\ 0.271\\ 0.370\\ 0.101\\ 0.237\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.394\\ 1.143\\ 4.179\\ 0.275\\ 0.330\\ 0.854\\ 3.546\\ 0.229\\ 0.255\\ 0.256\\ 0$	$\begin{array}{c} 0.695\\ 0.125\\ 0.110\\ 0.517\\ 0.584\\ 0.108\\ 0.096\\ 0.463\\ 0.507\\ 0.108\\ 0.507\\ 0.108\\ 0.509\\ \hline \end{array}$
n=150 n=200 n=250 n=50 n=100 n=150 n=200	$\begin{array}{c} \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ p \\ p \\ \alpha \\ \beta \\ \gamma \\ \gamma$	$\begin{array}{c} 0.232\\ 2.257\\ 2.831\\ 0.940\\ 0.219\\ 2.255\\ 2.808\\ 0.930\\ 0.215\\ 2.274\\ 2.789\\ 0.951\\ \hline \end{array}$	$\begin{array}{c} -0.163\\ 0.026\\ -0.011\\ -0.175\\ -0.098\\ -0.025\\ -0.002\\ -0.163\\ 0.003\\ -0.079\\ -0.033\\ 0.003\\ -0.189\\ \hline \alpha_0 = 1.2, \beta_0 =\\ \hline \text{Relative bias}\\ -0.072\\ -0.039\\ -0.142\\ -0.294\\ -0.039\\ -0.142\\ -0.294\\ -0.053\\ -0$	$\begin{array}{c} 0.018\\ 0.073\\ 0.094\\ 0.152\\ 0.013\\ 0.054\\ 0.073\\ 0.120\\ 0.051\\ 0.055\\ 0.143\\ 2, \gamma_0=5, p_1\\ \text{variance}\\ 00.313\\ 00.324\\ 04.127\\ 30.485\\ 00.131\\ 00.152\\ 01.237\\ 16.419\\ 00.076\\ 00.107\\ 00.706\\ 12.110\\ 0.052\\ 00.076\\ 00.00\\ 0.00$	$\begin{array}{c} 0.139\\ 0.276\\ 0.308\\ 0.414\\ 0.116\\ 0.0239\\ 0.271\\ 0.370\\ 0.237\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.370\\ 0.407\\ 0.236\\ 0.362\\ 0.394\\ 1.143\\ 0.362\\ 0.394\\ 1.143\\ 0.275\\ 0.330\\ 0.275\\ 0.330\\ 0.854\\ 3.546\\ 0.229\\ 0.277\\ 0$	$\begin{array}{c} 0.695\\ 0.125\\ 0.110\\ 0.517\\ 0.584\\ 0.108\\ 0.096\\ 0.463\\ 0.507\\ 0.108\\ 0.084\\ 0.509\\ \hline \end{array}$
n=150 n=200 n=250 n=50 n=100 n=150 n=200	$\begin{array}{c} \alpha \\ \beta \\ \gamma \\ p \\ \gamma \\ \gamma$	$\begin{array}{c} 0.232\\ 2.257\\ 2.831\\ 0.940\\ 0.219\\ 2.255\\ 2.808\\ 0.930\\ 0.215\\ 2.274\\ 2.789\\ 0.951\\ \hline 3.\\ \hline 3.\\$	$\begin{array}{c} -0.163\\ 0.026\\ -0.011\\ -0.175\\ -0.098\\ -0.025\\ -0.002\\ -0.163\\ -0.079\\ -0.033\\ 0.003\\ -0.189\\ \hline \\ \alpha_0 = 1.2, \beta_0 = \\ \hline \\ \mbox{Relative bias}\\ \hline \\ \alpha_0 = 1.2, \beta_0 = \\ \hline \\ \mbox{Relative bias}\\ -0.072\\ -0.039\\ -0.142\\ -0.294\\ -0.018\\ -0.028\\ -0.028\\ -0.053\\ -0.157\\ \hline \\ 0.004\\ -0.025\\ -0.030\\ -0.104\\ \hline \\ 0.008\\ -0.011\\ -0.025\\ -0.0011\\ -0.025\\ -0.068 \\ \hline \end{array}$	$\begin{array}{c} 0.018\\ 0.073\\ 0.094\\ 0.152\\ 0.013\\ 0.054\\ 0.073\\ 0.120\\ 0.010\\ 0.051\\ 0.055\\ 0.143\\ 2, \gamma_0 = 5, p,\\ variance\\ 00.313\\ 00.324\\ 04.127\\ 30.485\\ 00.131\\ 00.152\\ 01.1237\\ 16.419\\ 00.076\\ 00.107\\ 16.419\\ 00.076\\ 00.706\\ 12.110\\ 0.052\\ 00.076\\ 00.76\\ 00.484\\ o6 92e \end{array}$	$\begin{array}{c} 0.139\\ 0.276\\ 0.308\\ 0.414\\ 0.116\\ 0.0239\\ 0.271\\ 0.370\\ 0.101\\ 0.237\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.362\\ 0.394\\ 1.143\\ 4.179\\ 0.275\\ 0.330\\ 0.394\\ 1.143\\ 4.179\\ 0.275\\ 0.330\\ 0.854\\ 3.546\\ 0.229\\ 0.277\\ 0.706\\ 0.854\\ 3.546\\ 0.229\\ 0.277\\ 0.706\\ 0.854\\ 0.229\\ 0.277\\ 0.706\\ 0.854\\ 0.229\\ 0.277\\ 0.706\\ 0.854\\ 0.229\\ 0.277\\ 0.706\\ 0.858\\ 0.268\\ 0$	$\begin{array}{c} 0.695\\ 0.125\\ 0.110\\ 0.517\\ 0.584\\ 0.096\\ 0.463\\ 0.507\\ 0.108\\ 0.084\\ 0.509\\ \hline \\ \hline \\ Scaled RMSE\\ 0.472\\ 0.287\\ 0.430\\ 0.898\\ 0.302\\ 0.197\\ 0.228\\ 0.643\\ 0.229\\ 0.165\\ 0.191\\ 0.545\\ 0.191\\ 0.139\\ 0.141\\ 0.411\\ 0.411\\ \hline \end{array}$
n=150 n=200 n=250 n=50 n=100 n=150 n=200	$\begin{array}{c} \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ \gamma \\ \beta \\ \gamma \\ \gamma \\ \gamma \\ \gamma \\ \gamma \\ \gamma$	$\begin{array}{r} 0.232\\ 2.257\\ 2.831\\ 0.940\\ 0.219\\ 2.255\\ 2.808\\ 0.930\\ 0.215\\ 2.274\\ 2.789\\ 0.951\\ \hline 3.\\ Mean\\ 1.286\\ 2.078\\ 5.710\\ 8.412\\ 1.222\\ 2.058\\ 5.263\\ 7.522\\ 1.195\\ 2.049\\ 5.150\\ 7.178\\ 1.190\\ 2.022\\ 5.125\\ 6.943\\ 1.212\end{array}$	$\begin{array}{c} -0.163\\ 0.026\\ -0.011\\ -0.175\\ -0.098\\ -0.025\\ -0.002\\ -0.163\\ 0.003\\ -0.189\\ \hline \\ \alpha_0 = 1.2, \beta_0 =\\ \hline \\ \text{Relative bias}\\ \hline \\ -0.072\\ -0.039\\ -0.142\\ -0.072\\ -0.039\\ -0.142\\ -0.025\\ -0.053\\ -0.053\\ -0.053\\ -0.053\\ -0.157\\ \hline \\ 0.004\\ -0.025\\ -0.030\\ -0.104\\ \hline \\ 00.008\\ -0.011\\ -0.025\\ -0.068\\ -0.010\\ \hline \end{array}$	$\begin{array}{c} 0.018\\ 0.073\\ 0.094\\ 0.152\\ 0.013\\ 0.054\\ 0.073\\ 0.120\\ 0.010\\ 0.055\\ 0.143\\ 2, \gamma_0 = 5, p_1\\ \text{variance}\\ 00.313\\ 00.324\\ 04.127\\ 30.485\\ 00.313\\ 00.152\\ 01.237\\ 16.419\\ 00.076\\ 00.107\\ 00.766\\ 00.107\\ 00.766\\ 00.484\\ 06.926\\ 00.043\\ \end{array}$	$\begin{array}{c} 0.139\\ 0.276\\ 0.308\\ 0.414\\ 0.116\\ 0.0239\\ 0.271\\ 0.370\\ 0.101\\ 0.237\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.362\\ 0.394\\ 1.143\\ 4.179\\ 0.275\\ 0.330\\ 0.854\\ 3.546\\ 0.229\\ 0.277\\ 0.706\\ 2.668\\ 0.209\\ 0$	$\begin{array}{c} 0.695\\ 0.125\\ 0.110\\ 0.517\\ 0.584\\ 0.108\\ 0.096\\ 0.463\\ 0.507\\ 0.108\\ 0.507\\ 0.108\\ 0.509\\ \hline \end{array}$
n=150 n=200 n=250 n=50 n=100 n=150 n=200 n=250	$\begin{array}{c} \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ \gamma \\ \beta \\ \gamma \\ \gamma \\ \gamma \\ \gamma \\ \gamma \\ \gamma$	$\begin{array}{c} 0.232\\ 2.257\\ 2.831\\ 0.940\\ 0.219\\ 2.255\\ 2.808\\ 0.930\\ 0.215\\ 2.274\\ 4.789\\ 0.951\\ \hline \\ 3.\\ \hline \\ Mean\\ 1.286\\ 5.078\\ 5.710\\ 8.412\\ \hline \\ 1.222\\ 2.058\\ 5.710\\ 8.412\\ \hline \\ 1.222\\ 2.058\\ 5.263\\ 7.522\\ \hline \\ 1.195\\ 2.049\\ 5.150\\ 7.178\\ \hline \\ 1.190\\ 2.022\\ 5.125\\ 6.943\\ \hline \\ 1.212\\ 2.023\\ \hline \end{array}$	$\begin{array}{c} -0.163\\ 0.026\\ -0.011\\ -0.175\\ -0.098\\ -0.025\\ -0.002\\ -0.163\\ 0.003\\ -0.189\\ \hline mutual math math math math math math math math$	$\begin{array}{c} 0.018\\ 0.073\\ 0.094\\ 0.152\\ 0.013\\ 0.054\\ 0.073\\ 0.120\\ 0.010\\ 0.051\\ 0.055\\ 0.143\\ 2, \gamma_0 = 5, p\\ \text{variance}\\ 0.313\\ 00.324\\ 04.127\\ 16.419\\ 00.152\\ 01.237\\ 16.419\\ 00.076\\ 00.107\\ 00.706\\ 12.110\\ 0.052\\ 00.076\\ 00.107\\ 00.706\\ 00.107\\ 00.706\\ 00.107\\ 00.076\\ 00.107\\ 00.0060\\ 00.484\\ 06.926\\ 00.043\\ 00.060\\ \end{array}$	$\begin{array}{c} 0.139\\ 0.276\\ 0.308\\ 0.414\\ 0.128\\ 0.239\\ 0.271\\ 0.308\\ 0.271\\ 0.239\\ 0.271\\ 0.239\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.574\\ 2.152\\ 5.843\\ 0.394\\ 1.143\\ 4.179\\ 0.275\\ 0.394\\ 1.143\\ 4.179\\ 0.275\\ 0.330\\ 0.854\\ 3.546\\ 0.229\\ 0.277\\ 0.706\\ 2.668\\ 0.209\\ 0.246\\ \end{array}$	$\begin{array}{c} 0.695\\ 0.125\\ 0.110\\ 0.517\\ 0.584\\ 0.108\\ 0.096\\ 0.463\\ 0.507\\ 0.108\\ 0.084\\ 0.509\\ \hline \end{array}$
n=150 n=200 n=250 n=50 n=100 n=150 n=200 n=250	$\begin{array}{c} \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ \gamma \\ \beta \\ \gamma \\ \gamma \\ \beta \\ \gamma \\ \gamma \\ \beta \\ \gamma \\ \gamma$	$\begin{array}{c} 0.232\\ 2.257\\ 2.831\\ 0.940\\ 0.219\\ 2.255\\ 2.808\\ 0.930\\ 0.215\\ 2.274\\ 2.789\\ 0.951\\ \hline \\ 3.\\ 0.951\\ \hline \\ 3$	$\begin{array}{c} -0.163\\ 0.026\\ -0.011\\ -0.175\\ -0.098\\ -0.025\\ -0.002\\ -0.163\\ 0.003\\ -0.189\\ \hline\\ \alpha_0 = 1.2, \beta_0 =\\ \hline\\ \text{Relative bias}\\ -0.072\\ -0.039\\ -0.142\\ -0.294\\ -0.018\\ -0.028\\ -0.028\\ -0.053\\ -0.157\\ 0.004\\ -0.025\\ -0.030\\ -0.104\\ -0.025\\ -0.030\\ -0.104\\ -0.025\\ -0.030\\ -0.104\\ -0.025\\ -0.030\\ -0.104\\ -0.025\\ -0.030\\ -0.104\\ -0.025\\ -0.068\\ -0.011\\ -0.025\\ -0.068\\ -0.011\\ -0.011\\ -0.011\\ -0.018\\ \end{array}$	$\begin{array}{c} 0.018\\ 0.073\\ 0.094\\ 0.152\\ 0.013\\ 0.054\\ 0.073\\ 0.120\\ 0.010\\ 0.051\\ 0.055\\ 0.143\\ 2, \gamma_0 = 5, p_1\\ \hline variance\\ 00.313\\ 00.324\\ 04.127\\ 30.485\\ 00.131\\ 00.152\\ 01.237\\ 16.419\\ 00.076\\ 00.107\\ 16.419\\ 00.076\\ 00.706\\ 12.110\\ 0.052\\ 00.076\\ 00.076\\ 00.484\\ 06.926\\ 00.043\\ 00.060\\ 00.422\\ \end{array}$	$\begin{array}{c} 0.139\\ 0.276\\ 0.308\\ 0.414\\ 0.116\\ 0.0239\\ 0.271\\ 0.370\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.574\\ 2.152\\ 5.843\\ 0.362\\ 0.394\\ 1.143\\ 4.179\\ 0.275\\ 0.394\\ 1.143\\ 4.179\\ 0.275\\ 0.394\\ 1.143\\ 4.179\\ 0.275\\ 0.394\\ 1.143\\ 4.179\\ 0.275\\ 0.394\\ 1.143\\ 4.179\\ 0.275\\ 0.394\\ 1.143\\ 4.179\\ 0.275\\ 0.394\\ 1.143\\ 4.179\\ 0.275\\ 0.394\\ 1.143\\ 4.179\\ 0.275\\ 0.394\\ 1.143\\ 4.179\\ 0.275\\ 0.394\\ 1.143\\ 4.179\\ 0.275\\ 0.394\\ 1.143\\ 4.179\\ 0.275\\ 0.394\\ 1.143\\ 4.179\\ 0.275\\ 0.394\\ 1.143\\ 4.179\\ 0.275\\ 0.394\\ 1.143\\ 4.152\\ 0.290\\ 0.246\\ 0.657\\ 0.246\\ 0.57\\ 0.246\\ 0.57\\ 0.246\\ 0.57\\ 0.246\\ 0.57\\ 0.246\\ 0.57\\ 0.246\\ 0.57\\ 0.246\\ 0.57\\ 0.246\\ 0.57\\ 0.246\\ 0.57\\ 0.246\\ 0.57\\ 0.246\\ 0.57\\ 0.246\\ 0.57\\ 0.246\\ 0.57\\ 0.246\\ 0.57\\ 0.246\\ 0.57\\ 0.246\\ 0.57\\ 0.246\\ 0.56\\ 0.266\\ 0.57\\ 0.246\\ 0.56\\ 0.57\\ 0.246\\ 0.56\\ 0.57\\ 0.246\\ 0.56\\ 0.57\\ 0.246\\ 0.56\\ 0.57\\ 0.246\\ 0.56\\ 0.57\\ 0.246\\ 0.56\\ 0.57\\ 0.246\\ 0.56\\ 0.57\\ 0.246\\ 0.56\\ 0.57\\ 0.246\\ 0.56\\ 0.57\\ 0.246\\ 0.56\\ 0.57\\ 0.246\\ 0.56\\ 0.57\\ 0.246\\ 0.56\\ 0.57\\ 0.246\\ 0.56\\ 0.57\\ 0.246\\ 0.56\\ 0.57\\ 0.246\\ 0.56\\ 0.57\\ 0.246\\ 0.56\\ 0.57\\ 0.246\\ 0.56\\ 0.56\\ 0.56\\ 0.56\\ 0.56\\ 0.56\\ 0.56\\ 0.56\\ 0.56\\ 0.56\\ 0.56\\ 0.56\\ 0.56\\ 0.56\\ 0.56\\ 0$	$\begin{array}{c} 0.695\\ 0.125\\ 0.110\\ 0.517\\ 0.584\\ 0.108\\ 0.096\\ 0.463\\ 0.507\\ 0.108\\ 0.507\\ 0.108\\ 0.509\\ \hline \end{array}$
n=150 n=200 n=250 n=50 n=100 n=150 n=200 n=250	$\begin{array}{c} \alpha \\ \beta \\ \gamma \\ p \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ p \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ p \\ \gamma \\ p \\ \beta \\ \gamma \\ p \\ \beta \\ \gamma \\ \gamma \\ p \\ \beta \\ \gamma \\ \gamma$	$\begin{array}{r} 0.232\\ 2.257\\ 2.831\\ 0.940\\ 0.219\\ 2.255\\ 2.808\\ 0.930\\ 0.215\\ 2.274\\ 3.\\ 0.951\\ 3.\\ 0.951\\ 3.\\ 0.951\\ 3.\\ 0.951\\ 3.\\ 0.951\\ 3.\\ 0.951\\ 3.\\ 0.951\\ 3.\\ 0.951\\ 3.\\ 0.951\\ 3.\\ 0.951\\ 3.\\ 0.951\\ 3.\\ 0.951\\ 3.\\ 0.951\\ 3.\\ 0.951\\ 3.\\ 0.951\\ 3.\\ 0.951\\ 3.\\ 0.951\\ 3.\\ 0.951\\ 3.\\ 0.943\\ 3.\\ 0.943\\ 3.\\ 0.943\\ 3.\\ 0.951\\ 3.\\ 0.951\\ 3.\\ 0.943\\ $	$\begin{array}{c} -0.163\\ 0.026\\ -0.011\\ -0.175\\ -0.098\\ -0.025\\ -0.002\\ -0.163\\ -0.079\\ -0.033\\ 0.003\\ -0.189\\ \hline \\ \hline \\ \alpha_0 = 1.2, \beta_0 = \\ \hline \\ \ \\ \hline \\ \ \\ \hline \\ \ \\ \ \\ \ \\ \ \\ \$	$\begin{array}{c} 0.018\\ 0.073\\ 0.094\\ 0.152\\ 0.013\\ 0.054\\ 0.073\\ 0.054\\ 0.073\\ 0.120\\ 0.010\\ 0.055\\ 0.143\\ 2, \gamma_0 = 5, p_{\rm variance}\\ 00.313\\ 00.324\\ 04.127\\ 30.485\\ 00.313\\ 00.152\\ 01.237\\ 16.419\\ 00.076\\ 00.107\\ 00.766\\ 00.107\\ 00.706\\ 12.110\\ 0.052\\ 00.076\\ 00.107\\ 00.706\\ 00.484\\ 06.926\\ 00.043\\ 00.060\\ 00.422\\ 06.256\\ \end{array}$	$\begin{array}{c} 0.139\\ 0.276\\ 0.308\\ 0.414\\ 0.116\\ 0.0239\\ 0.271\\ 0.370\\ 0.101\\ 0.237\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.574\\ 2.152\\ 5.843\\ 0.362\\ 0.574\\ 2.152\\ 0.394\\ 1.143\\ 4.179\\ 0.275\\ 0.330\\ 0.854\\ 3.546\\ 0.229\\ 0.277\\ 0.706\\ 2.668\\ 0.209\\ 0.246\\ 0.657\\ 2.559\\ \end{array}$	$\begin{array}{c} 0.695\\ 0.125\\ 0.110\\ 0.517\\ 0.584\\ 0.108\\ 0.096\\ 0.463\\ 0.507\\ 0.108\\ 0.084\\ 0.509\\ \hline \end{array}$
n=150 n=200 n=250 n=50 n=100 n=150 n=200 n=250 n=300	$\begin{array}{c} \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ \gamma$	$\begin{array}{r} 0.232\\ 2.257\\ 2.831\\ 0.940\\ 0.219\\ 2.255\\ 2.808\\ 0.930\\ 0.215\\ 2.274\\ 2.789\\ 0.951\\ \hline 3.\\ \hline Mean\\ 1.286\\ 2.078\\ 5.710\\ 8.412\\ 1.222\\ 2.058\\ 5.710\\ 8.412\\ 1.222\\ 2.058\\ 5.263\\ 7.522\\ 1.195\\ 2.049\\ 5.150\\ 7.178\\ 1.190\\ 2.022\\ 5.125\\ 6.943\\ 1.212\\ 5.094\\ 1.203\\ \hline .040\\ 1.203\\ \hline \end{array}$	$\begin{array}{c} -0.163\\ 0.026\\ -0.011\\ -0.175\\ -0.098\\ -0.025\\ -0.002\\ -0.163\\ 0.003\\ -0.189\\ \hline\\ \hline\\ \alpha_0=1.2, \beta_0=\\ \hline\\ \text{Relative bias}\\ -0.072\\ -0.039\\ -0.142\\ -0.072\\ -0.039\\ -0.142\\ -0.072\\ -0.039\\ -0.142\\ -0.028\\ -0.028\\ -0.028\\ -0.028\\ -0.018\\ -0.028\\ -0.018\\ -0.025\\ -0.004\\ -0.025\\ -0.030\\ -0.104\\ -0.025\\ -0.030\\ -0.011\\ -0.025\\ -0.068\\ -0.011\\ -0.025\\ -0.068\\ -0.010\\ -0.011\\ -0.018\\ -0.008\\ -0.008\\ -0.008\\ -0.002\\ \hline\end{array}$	$\begin{array}{c} 0.018\\ 0.073\\ 0.094\\ 0.152\\ 0.013\\ 0.054\\ 0.073\\ 0.120\\ 0.010\\ 0.051\\ 0.055\\ 0.143\\ \hline 2, \gamma_0 = 5, p\\ \hline variance\\ 0.313\\ 00.324\\ 04.127\\ 10.313\\ 00.324\\ 04.127\\ 10.313\\ 00.324\\ 04.127\\ 10.313\\ 00.324\\ 04.127\\ 10.313\\ 00.324\\ 04.127\\ 10.313\\ 00.324\\ 04.127\\ 10.313\\ 00.324\\ 04.127\\ 10.313\\ 00.324\\ 04.127\\ 10.313\\ 00.324\\ 04.127\\ 10.313\\ 00.324\\ 00.131\\ 00.152\\ 01.237\\ 16.419\\ 00.131\\ 00.152\\ 01.237\\ 16.419\\ 00.484\\ 06.926\\ 00.043\\ 00.060\\ 00.422\\ 06.256\\ 00.034\\ \hline \end{array}$	$\begin{array}{c} 0.139\\ 0.276\\ 0.308\\ 0.414\\ 0.116\\ 0.0239\\ 0.271\\ 0.370\\ 0.101\\ 0.237\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.574\\ 2.152\\ 5.843\\ 0.362\\ 0.394\\ 1.143\\ 4.179\\ 0.275\\ 0.330\\ 0.854\\ 3.546\\ 0.229\\ 0.277\\ 0.706\\ 2.668\\ 0.209\\ 0.246\\ 0.657\\ 2.559\\ 0.186\\ \end{array}$	$\begin{array}{c} 0.695\\ 0.125\\ 0.110\\ 0.517\\ 0.584\\ 0.108\\ 0.096\\ 0.463\\ 0.507\\ 0.108\\ 0.084\\ 0.509\\ \hline \end{array}$
n=150 n=200 n=250 n=50 n=100 n=150 n=200 n=250 n=300	$\begin{array}{c} \alpha\\ \beta\\ \gamma\\ p\\ \alpha\\ \beta\\ \gamma\\ p\\ p\\ \alpha\\ \beta\\ \gamma\\ p\\ \alpha\\ \beta\\ \beta\\ \gamma\\ p\\ \alpha\\ \beta\\ \beta\\ \gamma\\ p\\ \alpha\\ \beta\\ \gamma\\ \beta\\ \gamma\\ \beta\\ \gamma\\ \beta\\ \gamma\\ \gamma\\ \beta\\ \gamma\\ \gamma\\$	$\begin{array}{r} 0.232\\ 2.257\\ 2.831\\ 0.940\\ 0.219\\ 2.255\\ 2.808\\ 0.930\\ 0.215\\ 2.274\\ 3.\\ 0.951\\ \hline \\ 3.\\ $	$\begin{array}{c} -0.163\\ 0.026\\ -0.011\\ -0.175\\ -0.098\\ -0.025\\ -0.002\\ -0.163\\ 0.003\\ -0.189\\ \hline\\ \hline \alpha_0 = 1.2, \beta_0 =\\ \hline\\ \hline \text{Relative bias}\\ \hline\\ \hline \alpha_0 = 1.2, \beta_0 =\\ \hline\\ \hline \text{Relative bias}\\ -0.072\\ -0.039\\ -0.142\\ -0.294\\ -0.018\\ -0.028\\ -0.018\\ -0.028\\ -0.053\\ -0.157\\ \hline\\ 0.004\\ -0.025\\ -0.030\\ -0.104\\ -0.025\\ -0.030\\ -0.104\\ -0.025\\ -0.030\\ -0.104\\ -0.025\\ -0.008\\ \hline\\ -0.011\\ -0.025\\ -0.068\\ \hline\\ -0.011\\ -0.018\\ -0.083\\ -0.002\\ -0.002\\ -0.008\\ \hline\end{array}$	$\begin{array}{c} 0.018\\ 0.073\\ 0.094\\ 0.152\\ 0.013\\ 0.054\\ 0.073\\ 0.120\\ 0.051\\ 0.055\\ 0.143\\ 2, \gamma_0=5, p_0\\ \hline variance\\ 00.313\\ 00.324\\ 04.127\\ 30.485\\ 00.131\\ 00.152\\ 01.237\\ 16.419\\ 00.076\\ 00.107\\ 16.419\\ 00.076\\ 00.706\\ 12.110\\ 0.052\\ 00.076\\ 00.076\\ 00.076\\ 00.048\\ 00.043\\ 00.043\\ 00.048\\ 00.048\\ \end{array}$	$\begin{array}{c} 0.139\\ 0.276\\ 0.308\\ 0.414\\ 0.116\\ 0.0239\\ 0.271\\ 0.370\\ 0.237\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.364\\ 0.574\\ 2.152\\ 5.843\\ 0.362\\ 0.394\\ 1.143\\ 4.179\\ 0.275\\ 0.394\\ 1.143\\ 4.179\\ 0.275\\ 0.394\\ 1.143\\ 4.179\\ 0.275\\ 0.394\\ 1.143\\ 4.179\\ 0.275\\ 0.394\\ 1.143\\ 4.179\\ 0.275\\ 0.394\\ 1.143\\ 4.179\\ 0.275\\ 0.394\\ 1.143\\ 4.179\\ 0.275\\ 0.394\\ 1.143\\ 4.179\\ 0.275\\ 0.394\\ 1.143\\ 4.179\\ 0.275\\ 0.394\\ 1.143\\ 4.179\\ 0.275\\ 0.394\\ 1.143\\ 4.179\\ 0.275\\ 0.394\\ 1.143\\ 4.179\\ 0.275\\ 0.394\\ 1.143\\ 4.179\\ 0.255\\ 0.394\\ 1.143\\ 4.152\\ 0.255\\ 0.221\\ 1.143\\ 1$	$\begin{array}{c} 0.695\\ 0.125\\ 0.110\\ 0.517\\ 0.584\\ 0.108\\ 0.096\\ 0.463\\ 0.507\\ 0.108\\ 0.507\\ 0.108\\ 0.509\\ \hline \end{array}$
n=150 n=200 n=250 n=50 n=100 n=150 n=200 n=250 n=300	$\begin{array}{c} \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ \gamma \\ \beta \\ \gamma \\ \gamma \\ \beta \\ \gamma \\ \gamma \\ \beta \\ \gamma \\ \gamma$	$\begin{array}{r} 0.232\\ 2.257\\ 2.831\\ 0.940\\ 0.219\\ 2.255\\ 2.808\\ 0.930\\ 0.215\\ 2.274\\ 0.951\\ 3.\\ $	$\begin{array}{c} -0.163\\ 0.026\\ -0.011\\ -0.175\\ -0.098\\ -0.025\\ -0.002\\ -0.163\\ -0.079\\ -0.033\\ 0.003\\ -0.189\\ \hline \\ \hline$	$\begin{array}{c} 0.018\\ 0.073\\ 0.094\\ 0.152\\ 0.013\\ 0.054\\ 0.073\\ 0.054\\ 0.073\\ 0.055\\ 0.143\\ 2, \gamma_0 = 5, p_1\\ \hline variance\\ 00.313\\ 00.324\\ 04.127\\ 30.485\\ 00.313\\ 00.324\\ 04.127\\ 30.485\\ 00.131\\ 00.152\\ 01.237\\ 16.419\\ 00.076\\ 00.706\\ 12.110\\ 0.052\\ 01.237\\ 16.419\\ 00.076\\ 00.706\\ 12.110\\ 0.052\\ 00.076\\ 00.484\\ 06.926\\ 00.043\\ 00.043\\ 00.048\\$	$\begin{array}{c} 0.139\\ 0.276\\ 0.308\\ 0.414\\ 0.116\\ 0.0239\\ 0.271\\ 0.370\\ 0.237\\ 0.362\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.236\\ 0.407\\ 0.574\\ 2.152\\ 5.843\\ 0.362\\ 0.574\\ 2.152\\ 5.843\\ 0.364\\ 0.574\\ 2.152\\ 0.394\\ 1.143\\ 4.179\\ 0.275\\ 0.330\\ 0.229\\ 0.366\\ 0.657\\ 2.668\\ 0.229\\ 0.246\\ 0.657\\ 2.559\\ 0.186\\ 0.221\\ 0.559\\ 0.186\\ 0.221\\ 0.569\\ 0.559\\ 0$	$\begin{array}{c} 0.695\\ 0.125\\ 0.110\\ 0.517\\ 0.584\\ 0.108\\ 0.096\\ 0.463\\ 0.507\\ 0.108\\ 0.507\\ 0.108\\ 0.509\\ \hline \end{array}$

Table 4.	MLE's a	nd mean	square	error of	f four	parame	eters ($[\alpha, \beta, \gamma]$	γ and	p)	in t	\mathbf{the}	case
		of the	(P-A-L)) modifi	ied W	eibull d	listrib	ution					

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		4.	$\alpha_0 = 1.5. \beta_0 =$	$3.5, \gamma_0 = 5$	$p_0 = 7$	
		Mean	$a_{\rm s} = 1.0, \beta_{\rm b} =$ Relative bias	$\frac{10}{\text{variance}}$	RMSE	Scaled RMSE
n=50	α	1.643	-0.095	00.402	0.649	0.433
	β	3.837	-0.096	01.165	1.131	0.323
	γ	5.747	-0.149	03.624	2.045	0.409
n = 100	p	9.398	-0.343	29.794	5.962	0.852
n = 100	B	3.689	-0.053	00.204 00.420	0.439	0.193
	γ	5.277	-0.055	01.177	1.119	0.224
	p	8.740	-0.248	18.488	4.634	0.662
n = 150	α	1.561	-0.041	00.126	0.360	0.240
	β	3.656	-0.044	00.263	0.536	0.153
	γ	5.136	-0.027	00.263	0.536	0.153
000	p	8.401	-0.200	10.928	3.590	0.513
n=200	α_{a}	1.304	-0.043	04.706	2.170	1.736
	~	0.490	0.019	00.004	0.065	0.131
	p	1.184	-0.315	00.903	0.992	1.102
n = 250	$\hat{\alpha}$	1.343	-0.075	00.049	0.706	0.564
	β	2.478	0.008	00.747	0.865	0.346
	γ	0.490	0.019	00.001	0.045	0.090
	p	1.114	-0.237	00.511	0.746	0.829
n=300	α	1.317	-0.053	00.480	0.696	0.556
	13	2.495	-0.001	00.731	0.855	0.342
	p	1.089	-0.210	00.462	0.706	0.784
	P	5 0	$r_0 = 0.8$, $\beta_0 = 1$	$5. \gamma_0 = 4.5$	$n_0 = 2.5$	0.101
		Mean	$\frac{10}{\text{Relative bias}}$	$\frac{0, \ 0}{\text{variance}}$	$p_0 = 2.0$ RMSE	Scaled RMSE
n=50	α	0.933	-0.166	0.212	0.479	0.598
	B	1.519	-0.013	0.196	0.443	0.295
	γ	4.989	-0.108	2.046	1.512	0.335
	p	3.532	-0.412	5.242	2.511	1.004
n=100	α	0.853	-0.067	0.101	0.323	0.403
	β	1.537	-0.025	0.082	0.290	0.194
	γ	4.676	-0.039	0.689	0.848	0.188
n = 150	p	0.848	-0.271	0.074	0.277	0.765
m=100	B	1.516	-0.011	0.046	0.215	0.143
	2	4.621	-0.268	0.407	0.649	0.144
	p	3.035	-0.214	2.122	1.552	0.621
		Mean	Belative bias	variance	BMSE	Scaled BMSE
n = 200	a	0.853	-0.066	0.052	0.235	0.294
	B	1.533	-0.023	0.038	0.199	0.133
	2	4.563	-0.014	0.284	0.536	0.119
	p	3.062	-0.225	1.776	1.446	0.578
n=250	α	0.839	-0.049	0.043	0.212	0.265
	β	1.523	-0.015	0.033	0.183	0.122
	γ	4.540	-0.008	0.218	0.469	0.104
	p	2.949	-0.179	1.396	1.264	0.506
n=300	α	0.835	-0.044	0.039	0.202	0.253
	Þ	1.527	-0.018	0.025	0.162	0.108
	n	2 922	-0.168	1.265	1 201	0.480
	P	6	$\alpha_0 = 0.7 \beta_0 = 3$	1.200	$p_0 = 1.5$	0.100
		Mean	$\frac{\alpha_0}{\beta_0} = 0.7, \beta_0 = 3$	$\frac{1.0, \gamma_0}{2.0} = 4, j$	$\frac{p_0 = 1.5}{\text{BMSE}}$	Scaled BMSE
n = 50	a	0.861	-0.230	0.276	0.549	0.785
m=50	B	3.853	-0.101	1.099	1.106	0.316
	2	4.391	-0.097	1.759	1.382	0.346
	\dot{p}	2.213	-0.475	2.171	1.637	1.091
n=100	α	0.765	-0.093	0.105	0.330	0.472
	β	3.649	-0.043	0.344	0.605	0.173
	γ	4.135	-0.034	0.411	0.655	0.164
	p	1.871	-0.247	0.881	1.009	0.673
n = 150	α	0.762	-0.089	0.066	0.264	0.378
	ß	3.655	-0.044	0.219	0.494	0.141
	· Y	4.035	-0.256	0.247	1.018	0.125
	P	0.750	-0.072	0.051	0.232	0.331
n = 200	CV	0.100	0.012	0.001	0.419	0.119
n=200	$\frac{\alpha}{\beta}$	3.605	-0.030	0.164		
n=200	$\alpha \beta \gamma$	$3.605 \\ 4.054$	-0.030 -0.013	$0.164 \\ 0.187$	0.436	0.109
n=200	$\begin{array}{c} \alpha \\ \beta \\ \gamma \\ p \end{array}$	$3.605 \\ 4.054 \\ 1.816$	-0.030 -0.013 -0.211	$0.164 \\ 0.187 \\ 0.617$	$0.419 \\ 0.436 \\ 0.847$	$0.109 \\ 0.565$
n=200 n=250	$\begin{array}{c} \alpha \\ \beta \\ \gamma \\ p \\ \hline \alpha \end{array}$	$3.605 \\ 4.054 \\ 1.816 \\ 0.743$	$-0.030 \\ -0.013 \\ -0.211 \\ -0.062$	$\begin{array}{r} 0.164 \\ 0.187 \\ 0.617 \\ 0.042 \end{array}$	$\begin{array}{r} 0.419 \\ 0.436 \\ 0.847 \\ \hline 0.209 \end{array}$	$\begin{array}{r} 0.109 \\ 0.565 \\ 0.299 \end{array}$
n=200 n=250	$\begin{array}{c} \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \end{array}$	$3.605 \\ 4.054 \\ 1.816 \\ 0.743 \\ 3.587$	$ \begin{array}{r} -0.030 \\ -0.013 \\ -0.211 \\ -0.062 \\ -0.025 \end{array} $	$0.164 \\ 0.187 \\ 0.617 \\ 0.042 \\ 0.144$	$\begin{array}{r} 0.419 \\ 0.436 \\ 0.847 \\ \hline 0.209 \\ 0.389 \end{array}$	0.109 0.565 0.299 0.111
n=200 n=250	$\begin{array}{c} \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ \gamma \end{array}$	$3.605 \\ 4.054 \\ 1.816 \\ 0.743 \\ 3.587 \\ 4.039$	$ \begin{array}{r} -0.030 \\ -0.013 \\ -0.211 \\ -0.062 \\ -0.025 \\ -0.009 \\ \end{array} $	$\begin{array}{r} 0.164 \\ 0.187 \\ 0.617 \\ 0.042 \\ 0.144 \\ 0.146 \end{array}$	$\begin{array}{r} 0.413 \\ 0.436 \\ 0.847 \\ \hline 0.209 \\ 0.389 \\ 0.384 \end{array}$	0.109 0.565 0.299 0.111 0.096
n=200 n=250	$\begin{array}{c} \alpha \\ \beta \\ \gamma \\ p \\ \hline \alpha \\ \beta \\ \gamma \\ p \\ \end{array}$	$3.605 \\ 4.054 \\ 1.816 \\ 0.743 \\ 3.587 \\ 4.039 \\ 1.754 \\ \hline$	$\begin{array}{r} -0.030\\ -0.013\\ -0.211\\ \hline -0.062\\ -0.025\\ -0.009\\ -0.169\\ \end{array}$	$\begin{array}{c} 0.164 \\ 0.187 \\ 0.617 \\ 0.042 \\ 0.144 \\ 0.146 \\ 0.429 \end{array}$	$\begin{array}{c} 0.413\\ 0.436\\ 0.847\\ 0.209\\ 0.389\\ 0.384\\ 0.703\\ \end{array}$	$\begin{array}{c} 0.109 \\ 0.565 \\ 0.299 \\ 0.111 \\ 0.096 \\ 0.469 \end{array}$
n=200 n=250 n=300	$\begin{array}{c} \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \alpha \\ \alpha \\ \gamma \\ \alpha \\ \alpha \\ \alpha \\ \alpha \\ \alpha \\ \alpha$	$\begin{array}{r} 3.605 \\ 4.054 \\ 1.816 \\ 0.743 \\ 3.587 \\ 4.039 \\ 1.754 \\ 0.749 \\ 0.749 \\ 0.749 \\ 0.749 \end{array}$	$\begin{array}{r} -0.030\\ -0.013\\ -0.211\\ -0.062\\ -0.025\\ -0.009\\ -0.169\\ -0.071\\ -0.071\end{array}$	$\begin{array}{c} 0.164 \\ 0.187 \\ 0.617 \\ 0.042 \\ 0.144 \\ 0.146 \\ 0.429 \\ 0.037 \\ 0.121 \end{array}$	$\begin{array}{c} 0.416\\ 0.436\\ 0.847\\ 0.209\\ 0.389\\ 0.384\\ 0.703\\ 0.200\\ 0.200\\ 0.200\\ \end{array}$	$\begin{array}{c} 0.109\\ 0.565\\ 0.299\\ 0.111\\ 0.096\\ 0.469\\ 0.286\\ 0.286\end{array}$
n=200 n=250 n=300	$\begin{array}{c} \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ \rho \\ \beta \\ \beta$	$\begin{array}{c} 3.605 \\ 4.054 \\ 1.816 \\ 0.743 \\ 3.587 \\ 4.039 \\ 1.754 \\ 0.749 \\ 3.582 \\ 2.992 \end{array}$	$\begin{array}{c} -0.030\\ -0.013\\ -0.211\\ -0.062\\ -0.025\\ -0.009\\ -0.169\\ -0.071\\ -0.024\\ 0.002\\ \end{array}$	$\begin{array}{c} 0.164\\ 0.187\\ 0.617\\ 0.042\\ 0.144\\ 0.146\\ 0.429\\ 0.037\\ 0.121\\ 0.120\\ \end{array}$	$\begin{array}{c} 0.436\\ 0.847\\ 0.209\\ 0.389\\ 0.384\\ 0.703\\ 0.200\\ 0.358\\ 0.347\end{array}$	$\begin{array}{c} 0.109\\ 0.565\\ 0.299\\ 0.111\\ 0.096\\ 0.469\\ 0.286\\ 0.102\\ 0.087\end{array}$
n=200 n=250 n=300	$\begin{array}{c} \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ p \\ \alpha \\ \beta \\ \gamma \\ n \end{array}$	$\begin{array}{r} 3.605 \\ 4.054 \\ 1.816 \\ 0.743 \\ 3.587 \\ 4.039 \\ 1.754 \\ 0.749 \\ 3.582 \\ 3.992 \\ 1.800 \end{array}$	$\begin{array}{r} -0.030\\ -0.013\\ -0.211\\ \hline 0.062\\ -0.025\\ -0.009\\ -0.169\\ \hline -0.071\\ -0.024\\ 0.002\\ -0.201\\ \end{array}$	$\begin{array}{c} 0.164\\ 0.187\\ 0.617\\ 0.042\\ 0.144\\ 0.146\\ 0.429\\ 0.037\\ 0.121\\ 0.120\\ 0.458\end{array}$	$\begin{array}{c} 0.436\\ 0.847\\ 0.209\\ 0.389\\ 0.384\\ 0.703\\ 0.200\\ 0.358\\ 0.347\\ 0.741\\ \end{array}$	$\begin{array}{c} 0.109\\ 0.565\\ 0.299\\ 0.111\\ 0.096\\ 0.469\\ 0.286\\ 0.102\\ 0.087\\ 0.494\\ \end{array}$

5 Application for Real Data

These following data from Murthy et al. [8].

Data Set: Failure Times of 50 Components:

 $\begin{array}{l} 0.036,\ 0.058,\ 0.061,\ 0.074,\ 0.078,\ 0.086,\ 0.102,\ 0.103,\ 0.114,\ 0.116,\ 0.148,\ 0.183,\ 0.192,\ 0.254,\ 0.262,\\ 0.379,\ 0.381,\ 0.538,\ 0.570,\ 0.574,\ 0.590,\ 0.618,\ 0.645,\ 0.961,\ 1.228,\ 1.600,\ 2.006,\ 2.054,\ 2.804,\ 3.058,\\ 3.076,\ 3.147,\ 3.625,\ 3.704,\ 3.931,\ 4.073,\ 4.393,\ 4.534,\ 4.893,\ 6.274,\ 6.816,\ 7.896,\ 7.904,\ 8.022,\ 9.337,\\ 10.94,\ 11.02,\ 13.88,\ 14.73,\ 15.08\end{array}$

We estimate parameters for The (P-A-L) Modified Weibull Distribution by maximum likelihood estimation are given by

 $\hat{\alpha} = 0.433, \quad \hat{\beta} = 8.751 \times 10^{-12}, \quad \hat{\gamma} = 5.336, \quad \hat{p} = 23.425$

Now, we are studying goodness of fit statistics test by Chi-Square, where the following hypothesis is:

 H_0 : the data come from The (P-A-L) Modified Weibull Distribution. H_1 : the data does not come from The (P-A-L) Modified Weibull Distribution.

The data were classified in 6 class intervals as shown in Table 5.

Class intervals	O_i	E_i	$\frac{(O_i - E_i)^2}{E - i}$
0-	24	6.5022	47.088
1-	2	6.3215	2.9542
2-	3	6.0619	1.5466
3-	6	5.7015	0.0156
4-	4	5.2234	0.2865
5-	0	4.6263	4.6263
6-	2	3.9342	0.9509
7-	2	3.1979	0.4488
8-	1	2.4827	0.8855
9-	1	1.8466	0.3881
10-	1	1.3239	0.0793
11-	1	0.9218	0.0066
12-	0	0.6278	0.6278
13-	1	0.4209	0.7968
14-	1	0.2791	1.8623
15 - 16	1	0.1837	3.6285
Total			66.1919

Table 5. For goodness of fit test

The calculated statistic value of Chi-square test (66.1919) is less than the tabulated statistic value (66.3386). Therefore the null hypothesis do not reject that the data came from The (P-A-L) Modified Weibull Distribution. We use software Mathematica 10 to get the goodness of fit test.

6 Conclusions

In this paper new lifetime distribution is presented which called The (P-A-L) Modified Weibull Distribution (PMWD). The hazard shape for The (P-A-L) Modified Weibull Distribution has different types of shapes such as decreasing and increasing hazard function. For example, if $p \rightarrow 1$, $\beta=1$ and $\gamma=1$ then $h(x)=\alpha+1$ is constant, whereas if $p \rightarrow 1$ and $\beta=1$, then $h(x)=\alpha+\gamma x^{\gamma-1}$, which is increasing for $\gamma > 1$ and decreasing for $\gamma < 1$. The maximum likelihood estimates of the

parameters and their variance covariance matrix were derived using software Mathematica 10 for numerically solving equations have no analytical solutions. By results of the simulation study, show that the bias for any estimator is decreased when the sample size increases also the relative bias is decreased when the sample size increases. Mean square error (MSE), root mean square error, relative mean square error decrease when the sample size increases. By application on real data, we found that the $PMWD(\alpha, \beta, \gamma, p)$ fits the data.

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Competing Interests

Authors have declared that no competing interests exist.

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